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CATEGORY THEORY RESEARCH SEMINAR:

When do products commute with quotients?

Dr Michael Hoefnagel (Stellenbosch University)

DATE: Tuesday, 18 June 2024 | 12h10 – 13h00 SAST

VENUES:
- Room 2002, Mathematics and Industrial Psychology Building, Stellenbosch University
- Online

ABSTRACT

Given normal subgroups $N \lhd G$ and $M \lhd H$, the subgroup $N \times M$ is normal in $G \times H$ and there is a canonical isomorphism

$$(G \times H)/(N \times M) \approx (G/N) \times (H/M).$$

This commutativity of binary products with quotients is a well-known consequence of the first isomorphism theorem.

The same basic commutativity of products with quotients is a categorical property and is very common in algebra, holding in the categories of rings, semirings, modules, Lie algebras, Boolean algebras, lattices, and even monoids. This begs the question of what can be said, if anything, about categories in which products commute with quotients. The aim of this talk is to answer this question.

The first half of the talk aims to justify our focus on the more general formulation of the property, namely, the commutativity of finite products with coequalisers, and to give an equational characterisation of algebraic categories in which it holds. The second half of the talk discusses its consequences, particularly as they relate to centrality (of morphisms). In the category of monoids, a morphism $f : X \to Y$ is central if for every $x \in X$ and every $y \in Y$ we have $f(x) + y = y + f(x)$. The central morphisms $\mathsf{Z}(X, Y)$ from X to Y form a commutative monoid, which acts on $\hom(X, Y)$. Moreover, the central morphisms are "well-behaved" and form what is called the *additive core* [1] (for monoids). This additive core is present in any pointed category in which finite products commute with coequalisers.

BIOGRAPHY

Michael earned his PhD from Stellenbosch University (SU) in December 2018. Since January 2019 he has worked as a lecturer of mathematics at SU. His main research interest lies within the intersection of algebra and category theory, i.e., within categorical algebra. In particular, his work has focused on understanding various topics in universal algebra from a categorical point of view, and also to understand various topics in category theory from the algebraic point of view.

WHO SHOULD ATTEND?

All are welcome. It will be assumed that the audience is familiar with basic concepts of category theory.

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