Karol Życzkowski

On-line Workshop

to celebrate the 60th anniversary
of the paper on quantum maps by

E.C.G. Sudarshan, P.M. Mathews and J.Rau

14 - 15 October 2021

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

Cracow, Poland

NITheCS

National Institute for Theoretical and Computational Sciences

South Africa

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Stochastic Dynamics of Quantum-Mechanical Systems

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AND

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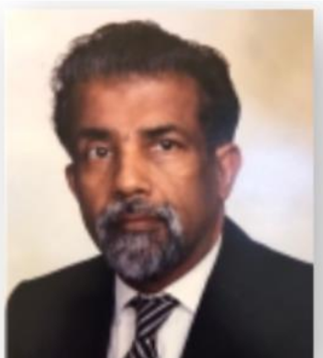
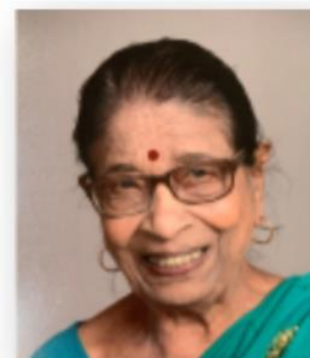
(Received August 15, 1960)

The most general dynamical law for a quantum mechanical system with a finite number of levels is formulated. A fundamental role is played by the so-called "dynamical matrix" whose properties are stated in a sequence of theorems. A necessary and sufficient criterion for distinguishing dynamical matrices corresponding to a Hamiltonian time-dependence is formulated. The non-Hamiltonian case is discussed in detail and the application to paramagnetic relaxation is outlined.



Ennackal Chandy George Sudarshan (1931 – 2018)

P.M. Mathews



Jayaseetha Rau

1960 - 1961 Quantum dynamics in discrete time I

Linear transformation of the density matrix ρ

$$\rho_{r,s}(t) = A_{rs,r's'}(t,t_0)\rho_{r's'}(t_0), \qquad (14)$$

***Dynamical matrix* B** obtained by **reshuffling** of the superoperator A

$$B_{rr',ss'} = A_{rs,r's'}. \qquad (14)$$

Properties of a **dynamical matrix** $B=A^R$

It immediately follows that B is Hermitian and positive semidefinite; we can rewrite (11′) and (12′) in the form:

$$B_{rr',ss'} = (B_{ss',rr'})^*, \quad \text{(Hermiticity)} \qquad (15)$$

$$z_{rr'}{}^* B_{rr',ss'} z_{ss'} \geqslant 0. \quad \text{(positive semidefiniteness)} \qquad (16)$$

The trace condition (13′) is still complicated and becomes

$$B_{rr',rs'} = \delta_{r's'}; \qquad (17)$$

1960 - 1961 **Quantum dynamics in discrete time II**

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Choi Theorem: if **B>0** then the map **A=B^R** is completely positive (1972)

Quantum dynamics in continuous time

V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, *Completely positive dynamical semigroups of N-level systems*, J. Math. Phys. **17**, 821 (1976).

G. Lindblad, *On the generators of quantum dynamical semigroups*, Commun. Math. Phys. **48**, 119 (1976).

GKLS equation

$$\mathcal{L}\rho = -i[H,\rho] + \frac{1}{2}\sum_j \left([V_j, \rho V_j^*] + [V_j\rho, V_j^*]\right).$$

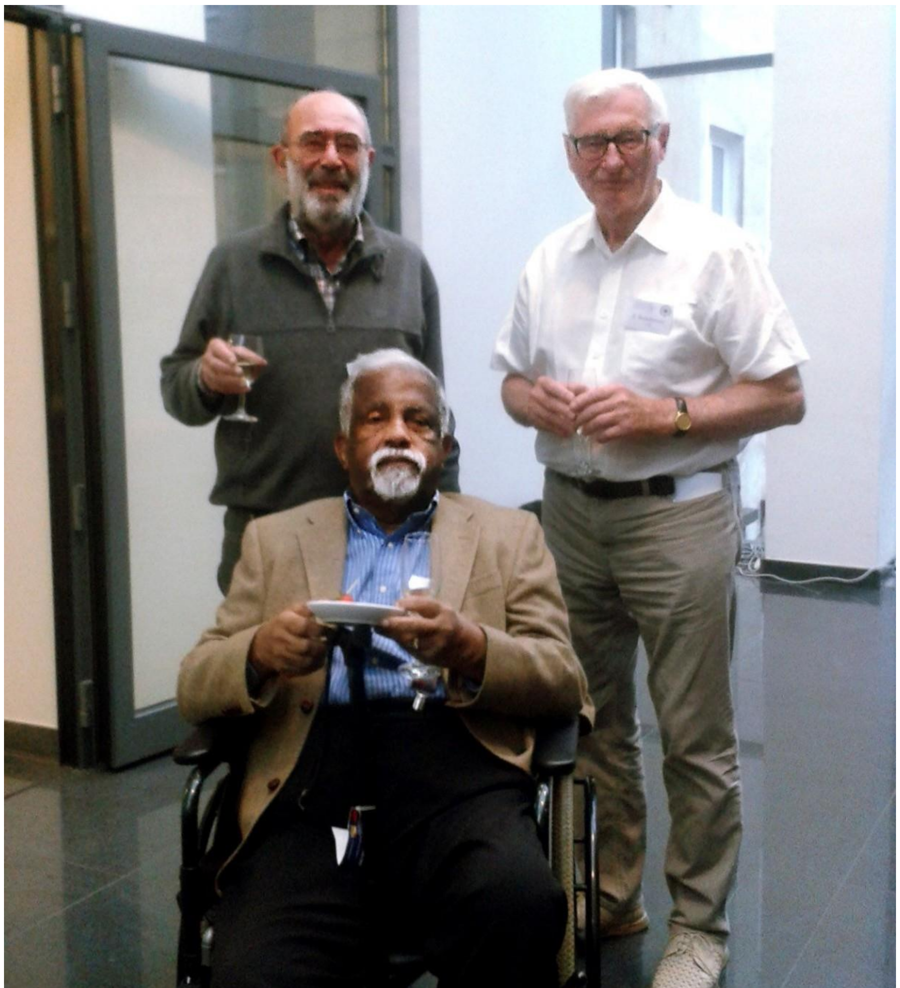
Figure 1: Picture taken in Prof. Ingarden's office (December 1975). From left to right: Roman Ingarden, Andrzej Kossakowski, George Sudarshan and Vittorio Gorini.

48 Symposium on Mathematical Physics

Gorini-Kossakowski-Lindblad-Sudarshan

Master Equation - **40 Years After**

Toruń, June 10 - 12, **2016**



Vittorio Gorini

Andrzej Kossakowski

George Sudarshan

(2016)

Guests of Honour:

P. M. Mathews and Jayaseetha Rau

(co-authors of the paper)

G. Bhamathi and Ashok Sudarshan

(family of **George Sudarshan**)

Enjoy the workshop !

Thursday, Oct. 14, 3.00 pm – 7.00 pm (CET)
Friday, Oct. 15, 11.00 am. – 4.00 pm (CET)
(mind a new link for tomorrow !)