Open spin chain asymptotic states

Fabio Benatti

Department of Physics, University of Trieste, Italy

"Stochastic Dynamics of Quantum-Mechanical Systems"
14-15 October 2021

Joint work with R. Floreanini and L. Memarzadeh

Outline I

1. **Open quantum spin chain**
2. **Master equation**
3. **Asymptotic States**

Weak-coupling limit: still a source of puzzling effects

- Global Hamiltonian: $H_\lambda(g) = H_S(g) + H_{env} + \lambda\, H_{int}$
- Weak-coupling limit: $\lambda \to 0$, $t \to \infty$, so that $\lambda^2\, t = \tau$ fixed
- Reduced dynamics:

$$\rho_S(t) = w - \lim_{\lambda, t} \mathrm{Tr}_{env}\left(\mathrm{e}^{-i\,t\,H_\lambda(g)}\,\rho_S \otimes \rho_{env}\,\mathrm{e}^{i\,t\,H_\lambda(g)}\right)$$

Weak-coupling limit: still a source of puzzling effects

- Global Hamiltonian: $H_\lambda(g) = H_S(g) + H_{env} + \lambda\, H_{int}$
- Weak-coupling limit: $\lambda \to 0$, $t \to \infty$, so that $\lambda^2\, t = \tau$ fixed
- Reduced dynamics:

$$\rho_S(t) = w - \lim_{\lambda,t} \mathrm{Tr}_{env}\left(\mathrm{e}^{-i\,t\,H_\lambda(g)}\,\rho_S \otimes \rho_{env}\,\mathrm{e}^{i\,t\,H_\lambda(g)}\right)$$

Issue: exchangeability of limits

$$\boxed{w - \lim_{\lambda,t}\,\lim_{g\to g^*} \overset{?}{=} \lim_{g\to g^*}\; w - \lim_{\lambda,t}}$$

XX spin-chain with open boundary conditions

$$H = \Delta \sum_{\ell=1}^{N} \sigma_z^{(\ell)} + g \sum_{\ell=1}^{N-1} \left(\sigma_x^{(\ell)} \sigma_x^{(\ell+1)} + \sigma_y^{(\ell)} \sigma_y^{(\ell+1)} \right)$$

$$= \boxed{\Delta \sum_{\ell=1}^{N} \sigma_z^{(\ell)} + 2g \sum_{\ell=1}^{N-1} \left(\sigma_+^{(\ell)} \sigma_-^{(\ell+1)} + \sigma_-^{(\ell)} \sigma_+^{(\ell+1)} \right)}$$

- Jordan-Wigner fermionization:

$$\boxed{c_j = \prod_{k=1}^{j-1} (-\sigma_z^{(k)})\, \sigma_-^{(j)}} \ , \quad \left\{ c_j \, , \, c_k^\dagger \right\} = \delta_{jk}$$

$$\boxed{\sigma_-^{(j)} = \prod_{k=1}^{j-1} \left(1 - 2\, c_j^\dagger\, c_j \right) c_j}$$

- Bogoljubov transformation:

$$\boxed{b_\ell = \sum_{j=1}^{N} u_{\ell j}\, c_j}\ , \quad u_{\ell k} = \sqrt{\frac{2}{N+1}}\sin\left(\frac{\ell k \pi}{N+1}\right)$$

$$\boxed{\left\{b_j\,,\, b_\ell^\dagger\right\} = \delta_{j\ell}}\ , \quad \boxed{c_j = \sum_{\ell=1}^{N} u_{j\ell}\, b_\ell}$$

Diagonal Hamiltonian

$$\boxed{H_N = \sum_{\ell=1}^{N} \omega_\ell\, b_\ell^\dagger\, b_\ell}\ , \quad \boxed{\omega_\ell = 2\,\Delta + 4\, g\, \cos\left(\frac{\ell\pi}{N+1}\right)}$$

Eigenfrequencies

- decrease with N:

$$\begin{aligned}\omega_1 &= 2\,\Delta + 4\,g\,\cos\frac{\pi}{N+1} \geq \omega_2 \geq \\ &\geq \omega_3 \geq \quad \ldots \quad \geq \omega_N = 2\,\Delta - 4\,g\,\cos\frac{\pi}{N+1}\end{aligned}$$

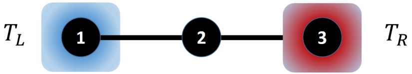
- All positive when $0 \leq \ell \leq \dfrac{N+1}{2}$ and

$$\boxed{\omega_N = 2\,\Delta - 4\,g\,\cos\frac{\pi}{N+1} \geq 0 \Longleftrightarrow g \leq \frac{\Delta}{2\cos\frac{\pi}{N+1}}}$$

- $\omega_{p-1} \geq 0$ but $0 \geq \omega_p \geq \ldots \geq \omega_N$ when $p > \dfrac{N+1}{2}$ and

$$\boxed{\frac{\Delta}{2\left|\cos\frac{\pi\,(p-1)}{N+1}\right|} \geq g > \frac{\Delta}{2\left|\cos\frac{\pi\,p}{N+1}\right|}}$$

Open quantum spin chain



- Chain ends ($\alpha = L, R$) coupled to Bosonic thermal baths

$$H_{int} = \lambda \sum_{\alpha=L,R} \left(\sigma_+^{(\alpha)} \mathfrak{B}_\alpha + \sigma_-^{(\alpha)} \mathfrak{B}_\alpha^\dagger \right)$$

$$\sigma_-^{(L)} = \sigma_-^{(1)} = \sum_{j=1}^{N} u_{1j}\, b_j \quad , \quad \sigma_-^{(R)} = \sigma_-^{(N)} = (-1)^{\sum_{\ell=1}^{N} b_\ell^\dagger b_\ell} \sum_{j=1}^{N} u_{Nj} b_j$$

Thermal baths

- Bath operators

$$\mathfrak{B}_\alpha = \int_0^\infty d\nu \, h_\alpha(\nu) \, \mathfrak{a}_\alpha(\nu) \,, \quad h^*_\alpha(\nu) = h_\alpha(\nu)$$

$$\left[\mathfrak{a}_\alpha(\nu), \, \mathfrak{a}^\dagger_{\alpha'}(\nu')\right] = \delta_{\alpha,\alpha'} \, \delta(\nu - \nu')$$

- Bath thermal states at inverse temperatures $\beta_{L,R}$:

$$\rho_{env} = \frac{e^{-\beta_L H_L}}{\mathrm{Tr}\left(e^{-\beta_L H_L}\right)} \otimes \frac{e^{-\beta_R H_R}}{\mathrm{Tr}\left(e^{-\beta_R H_R}\right)}$$

$$\boxed{H_\alpha = \int_0^{+\infty} d\nu \, \nu \, \mathfrak{a}^\dagger_\alpha(\nu) \, \mathfrak{a}_\alpha(\nu)}$$

- Thermal expectations:

$$\mathrm{Tr}_B\Big(\rho_{env}\,\mathfrak{a}^\dagger_\alpha(\nu)\mathfrak{a}_{\alpha'}(\nu')\Big) = \delta_{\alpha\alpha'}\delta(\nu-\nu')\,n_\alpha(\nu)$$

$$\mathrm{Tr}_B\Big(\rho_{env}\,\mathfrak{a}_\alpha(\nu)\,\mathfrak{a}^\dagger_{\alpha'}(\nu')\Big) = \delta_{\alpha\alpha'}\delta(\nu-\nu')\,(1+n_\alpha(\nu))$$

$$n_\alpha(\nu) = \frac{1}{e^{\beta_\alpha \nu}-1}\ , \qquad \boxed{\nu \geq 0}\ .$$

- Thermal expectations:

$$\mathrm{Tr}_B\Big(\rho_{env}\, \mathfrak{a}^\dagger_\alpha(\nu)\mathfrak{a}_{\alpha'}(\nu')\Big) = \delta_{\alpha\alpha'}\delta(\nu-\nu')\, n_\alpha(\nu)$$

$$\mathrm{Tr}_B\Big(\rho_{env}\, \mathfrak{a}_\alpha(\nu)\, \mathfrak{a}^\dagger_{\alpha'}(\nu')\Big) = \delta_{\alpha\alpha'}\delta(\nu-\nu')\,(1+n_\alpha(\nu))$$

$$n_\alpha(\nu) = \frac{1}{e^{\beta_\alpha \nu}-1}\ , \qquad \boxed{\nu \geq 0}\ .$$

Counter-rotating spin-bath couplings:

$$\boxed{H_{int} = \lambda \sum_{\alpha=L,R}\Big(\sigma_+^{(\alpha)}\mathfrak{B}_\alpha + \sigma_-^{(\alpha)}\mathfrak{B}^\dagger_\alpha\Big)}$$

$$\sigma_+^{(\alpha)}\ \mathfrak{a}_\alpha(\nu)\ , \qquad \sigma_-^{(\alpha)}\ \mathfrak{a}^\dagger_\alpha(\nu)$$

Setting

$$H = H_N(g) + \sum_{\alpha=R,L} H_\alpha + H_{int} \quad , \quad H_\alpha = \int_0^{+\infty} \mathrm{d}\nu \, \nu \, \mathfrak{a}^\dagger_\alpha(\nu) \, \mathfrak{a}_\alpha(\nu)$$

$$H_N(g) = \sum_{\ell=1}^{N} \omega_\ell \, b^\dagger_\ell \, b_\ell \quad , \quad \omega_\ell = 2\,\Delta + 4\,g\,\cos\left(\frac{\ell\pi}{N+1}\right)$$

$$H_{int} = \lambda \sum_{\alpha=L,R} \sum_{\ell=1}^{N} u_{\alpha\ell}\left(b^\dagger_{\alpha\ell}\mathfrak{B}_\alpha + b_{\alpha\ell}\mathfrak{B}^\dagger_\alpha\right)$$

$$b_{\alpha\ell} = \begin{cases} b_\ell & \dots \quad \alpha = L \\ (-)^{\sum_{k=1}^{N} b^\dagger_k b_k} \, b_\ell & \dots \quad \alpha = R \end{cases}$$

Weak-Coupling Limit: $\lambda \to 0$, $\lambda^2\, t = \tau$

- Initial state: $\rho_{\mathrm{tot}}(0) = \rho \otimes \rho_{env}$

Master equation: $$\boxed{\partial_t \rho_t = -i\Big[H_N(g)\,,\,\rho_t\Big] + \lambda^2\,\widetilde{\mathbb{L}}[\rho_t]}$$

Weak-Coupling Limit: $\lambda \to 0$, $\lambda^2\, t = \tau$

- Initial state: $\rho_{\rm tot}(0) = \rho \otimes \rho_{env}$

Master equation: $\boxed{\partial_t \rho_t = -i\Big[H_N(g)\,,\,\rho_t\Big] + \lambda^2\,\widetilde{\mathbb{L}}[\rho_t]}$

$$\widetilde{\mathbb{L}}[\rho] = -\boxed{\lim_{T\to+\infty}\frac{1}{T}\int_0^T \mathrm{d}s}$$

Weak-Coupling Limit: $\lambda \to 0$, $\lambda^2 t = \tau$

- Initial state: $\rho_{\rm tot}(0) = \rho \otimes \rho_{env}$

Master equation: $\boxed{\partial_t \rho_t = -i\Big[H_N(g)\,,\,\rho_t\Big] + \lambda^2 \widetilde{\mathbb{L}}[\rho_t]}$

$$\widetilde{\mathbb{L}}[\rho] = -\boxed{\lim_{T\to+\infty} \frac{1}{T}\int_0^T {\rm d}s}\,\boxed{\int_0^{+\infty} {\rm d}t} \times$$

$$\times {\rm Tr}_B\left(\Big[\mathbf{H}_{\mathbf{int}}(t,s)\,,\,\Big[\mathbf{H}_{\mathbf{int}}(0,s)\,,\,\rho\otimes\rho_{env}\Big]\Big]\right)$$

$$\mathbf{H_{int}}(t,s) = \mathrm{e}^{i(t+s)\mathbf{H_N}(\mathbf{g})}\,\mathrm{e}^{it(\mathbf{H_L}+\mathbf{H_R})}\,\mathbf{H_{int}}\,\mathrm{e}^{-it(\mathbf{H_L}+\mathbf{H_R})}\,\mathrm{e}^{-i(t+s)\mathbf{H_N}(\mathbf{g})}$$

$$= \sum_{\alpha=L,R}\sum_{\ell=1}^{N} u_{\alpha\ell}\int_0^{+\infty}\mathrm{d}\nu\, h_\alpha(\nu)\,\left(\mathrm{e}^{it(\omega_\ell-\nu)}\,\mathrm{e}^{i\,s\,\omega_\ell}\, b^\dagger_{\alpha\ell}\,\mathfrak{a}_\alpha(\nu) + h.c.\right)$$

$$\mathbf{H}_{\mathbf{int}}(t,s) = \mathrm{e}^{i(t+s)\mathbf{H}_{\mathbf{N}}(\mathbf{g})}\, \mathrm{e}^{it(\mathbf{H}_{\mathbf{L}}+\mathbf{H}_{\mathbf{R}})}\, \mathbf{H}_{\mathbf{int}}\, \mathrm{e}^{-it(\mathbf{H}_{\mathbf{L}}+\mathbf{H}_{\mathbf{R}})}\, \mathrm{e}^{-i(t+s)\mathbf{H}_{\mathbf{N}}(\mathbf{g})}$$

$$= \sum_{\alpha=L,R} \sum_{\ell=1}^{N} u_{\alpha\ell} \int_0^{+\infty} \mathrm{d}\nu\, h_\alpha(\nu) \left(\mathrm{e}^{it(\omega_\ell-\nu)}\, \mathrm{e}^{i\,s\,\omega_\ell}\, b^\dagger_{\alpha\ell}\, \mathfrak{a}_\alpha(\nu) + h.c. \right)$$

$$\mathbf{H}_{\mathbf{int}}(0,s) = \mathrm{e}^{i\,s\,\mathbf{H}_{\mathbf{N}}(\mathbf{g})}\, \mathbf{H}_{\mathbf{int}}\, \mathrm{e}^{-i\,s\,\mathbf{H}_{\mathbf{N}}(\mathbf{g})}$$

$$= \sum_{\alpha=L,R} \sum_{\ell=1}^{N} u_{\alpha\ell} \int_0^{+\infty} \mathrm{d}\nu\, h_\alpha(\nu) \left(\mathrm{e}^{i\,s\,\omega_\ell}\, b^\dagger_{\alpha\ell}\, \mathfrak{a}_\alpha(\nu) + h.c. \right)$$

Contributions from $\mathrm{Tr}_B\left(\left[\mathbf{H_{int}}(t,s)\,,\,\left[\mathbf{H_{int}}(0,s)\,,\,\rho\otimes\rho_{env}\right]\right]\right)$

- $\boxed{\int_0^{+\infty}\mathrm{d}t\,\mathrm{e}^{it(\omega_\ell-\nu)}}$: $\pi\,\delta(\omega_\ell-\nu)+i\,P.V.\frac{1}{\omega_\ell-\nu}$
- Ergodic average $\boxed{\lim_{T\to+\infty}\frac{1}{T}\int_0^T\mathrm{d}s\,\mathrm{e}^{i\,s(\omega_\ell-\omega_{\ell'})}}$: $\omega_\ell=\omega_{\ell'}\Leftrightarrow\ell=\ell'$
- Thermal expectations: $\alpha=\alpha'$, $\nu=\nu'$,

$$\mathrm{Tr}_B\Big(\rho_{env}\,\mathfrak{a}_\alpha(\nu)\,\mathfrak{a}^\dagger_\alpha(\nu)\Big)\ ,\quad \mathrm{Tr}_B\Big(\rho_{env}\,\mathfrak{a}^\dagger_\alpha(\nu)\,\mathfrak{a}_\alpha(\nu)\Big)$$

Global vs Local approach

- Global approach: $H_N(g)$ with $g \neq 0$
- Local approach: set $g = 0$ and change $H_N(0)$ to $H_N(g)$ after the weak-coupling limit

Global vs Local approach

- Global approach: $H_N(g)$ with $g \neq 0$
- Local approach: set $g = 0$ and change $H_N(0)$ to $H_N(g)$ after the weak-coupling limit

In the following: global approach

F.B., R. Floreanini, L. Memarzadeh, PRX Quantum 2, 030344 (2021)

GKSL master equation: $\widetilde{\mathbb{L}}[\rho] = -i\,[H_{LS}\,,\rho\,] + \mathbb{D}[\rho]$

$$\frac{\partial \rho_t}{\partial t} = -i\big[H_N(g) + \lambda^2\, H_{LS}\,,\ \rho_t\big] + \lambda^2 \mathbb{D}[\rho(t)] = \mathbb{L}[\rho_t]$$

- Lamb-shift correction: all frequencies contribute

$$H_{LS} = \sum_{\ell=1}^{N} \underbrace{\sum_{\alpha=L,R} u^2_{\alpha\ell} \left(P.V. \int_0^{+\infty} \frac{h^2_\alpha(\nu)(1+2n_\alpha(\nu))}{\omega_\ell - \nu} \right)}_{h_\ell} \underbrace{b^\dagger_{\alpha\ell}\, b_{\alpha\ell}}_{b^\dagger_\ell\, b_\ell}$$

$$= \lambda^2 \sum_{\ell=1}^{N} h_\ell\, b^\dagger_\ell\, b_\ell$$

- Dissipative correction: only positive frequencies contribute:

$$\mathbb{D}[\rho] = \sum_{\ell=1}^{N} \mathbb{D}_\ell[\rho]$$

$$\mathbb{D}_\ell[\rho] = \sum_{\alpha=L,R} 2\pi\, h_\alpha^2(\omega_\ell)\, u_{\alpha\ell}^2 \boxed{\Theta(\omega_\ell)} \times$$

$$\times \left\{\Big(1+n_\alpha(\omega_\ell)\Big)\left(b_{\alpha\ell}\,\rho\, b_{\alpha\ell}^\dagger - \frac{1}{2}\left\{b_{\alpha\ell}^\dagger\, b_{\alpha\ell}\,,\rho\right\}\right)\right\}$$

$$\times \left\{n_\alpha(\omega_\ell)\left(b_{\alpha\ell}^\dagger\,\rho\, b_{\alpha\ell} - \frac{1}{2}\left\{b_{\alpha\ell}\, b_{\alpha\ell}^\dagger\,,\rho\right\}\right)\right\}$$

Generator

$$\mathbb{L}[\rho] = \sum_{\ell=1}^{N} \mathbb{L}_\ell[\rho]$$

$$\mathbb{L}_\ell[\rho] = -i\left[(\omega_\ell + \lambda^2 h_\ell)\, b_\ell^\dagger\, b_\ell\,,\,\rho\right] + \lambda^2\, \mathbb{D}_\ell[\rho]$$

Generator

$$\mathbb{L}[\rho] = \sum_{\ell=1}^{N} \mathbb{L}_\ell[\rho]$$

$$\mathbb{L}_\ell[\rho] = -i\left[(\omega_\ell + \lambda^2 h_\ell)\, b_\ell^\dagger\, b_\ell\,,\,\rho\right] + \lambda^2\, \mathbb{D}_\ell[\rho]$$

Open chain dynamics:

$$\mathbb{L}_\ell \mathbb{L}_k = \mathbb{L}_k \mathbb{L}_\ell \Longrightarrow \gamma_t = \mathrm{e}^{t\,\mathbb{L}} = \prod_{\ell=1}^{N} \mathrm{e}^{t\,\mathbb{L}_\ell}$$

Stationary states of quantum dynamical semigroups

A. Frigerio, Commun. Math. Phys. 63 (1978)

Given a generator

$$\mathbb{L}[\rho] = -i[H\,,\,\rho] + \sum_k \left(V_k\,\rho\,V_k^\dagger - \frac{1}{2}\left\{ V_k^\dagger\,V_k\,,\,\rho \right\} \right)$$

seek the commutant $\mathcal{C}$ of the set $\left\{ H, V_k, V_k^\dagger \right\}$.

Stationary states of quantum dynamical semigroups

A. Frigerio, Commun. Math. Phys. 63 (1978)

Given a generator

$$\mathbb{L}[\rho] = -i[H\,,\,\rho] + \sum_k \left(V_k\,\rho\,V_k^\dagger - \frac{1}{2}\left\{V_k^\dagger\,V_k\,,\,\rho\right\}\right)$$

seek the commutant $\mathcal{C}$ of the set $\left\{H, V_k, V_k^\dagger\right\}$.

if $\mathcal{C} = \{1\}$ then there is only one stationary state such that

$$\mathbb{L}[\rho_\infty] = 0$$

if $\mathcal{C}$ is commutative, generated by orthogonal projections

$$P_n P_m = \delta_{nm} P_n \ , \quad \sum_n P_n = 1 \ ,$$

given a faithful stationary state ρ_∞, then the stationary states form a convex set:

$$\rho_\infty^{\{\lambda_n\}} = \sum_n \lambda_n \frac{P_n \rho_\infty P_n}{\mathrm{Tr}(\rho_\infty P_n)} \ , \quad \lambda_n = \mathrm{Tr}(\rho P_n) \ ,$$

where ρ is any initial state.

if $\mathcal{C}$ is commutative, generated by orthogonal projections

$$P_n\,P_m = \delta_{nm}\,P_n\ ,\quad \sum_n P_n = 1\ ,$$

given a faithful stationary state ρ_∞, then the stationary states form a convex set:

$$\rho_\infty^{\{\lambda_n\}} = \sum_n \lambda_n\,\frac{P_n\rho_\infty\,P_n}{\mathrm{Tr}(\rho_\infty\,P_n)}\ ,\quad \lambda_n = \mathrm{Tr}(\rho\,P_n)\ ,$$

where ρ is any initial state.

Proof:

$$\mathcal{C} = \{P_n\}_n\,\&\,\mathbb{L}[\rho_\infty] = 0 \Longrightarrow \mathbb{L}[P_n\,\rho_\infty\,P_n] = P_n\,\mathbb{L}[\rho_\infty]\,P_n = 0$$

Result 1

All frequencies $\omega_\ell \geq 0 \quad \Longrightarrow \quad$ all $b_\ell^\dagger b_\ell, b_{\alpha\ell}, b_{\alpha\ell}^\dagger$ contribute
$\Longrightarrow$ commutant $\mathcal{C} = \{1\} \quad \Longrightarrow \quad$ unique stationary state ρ_∞:

$$\rho_\infty = \prod_{\ell=1}^{N} \left(\mu_\ell + \nu_\ell \, b_\ell^\dagger \, b_\ell\right) \Longrightarrow \mathbb{L}_k[\rho_\infty] = 0$$

$$\mu_\ell = \frac{C_\ell}{C_\ell + \widetilde{C}_\ell} \qquad \nu_\ell = \frac{\widetilde{C}_\ell - C_\ell}{C_\ell + \widetilde{C}_\ell}$$

$$C_\ell = 2\pi \sum_{\alpha=L,R} u_{\alpha\ell}^2 \, h_\alpha^2(\omega_\ell) \, (1 + n_\alpha(\omega_\ell))$$

$$\widetilde{C}_\ell = 2\pi \sum_{\alpha=L,R} u_{\alpha\ell}^2 \, h_\alpha^2(\omega_\ell) \, n_\alpha(\omega_\ell)$$

Result 2

Coupling g such that only frequencies $\omega_1 \geq \omega_2 \geq \ldots \omega_{p-1} \geq 0$

$\Longrightarrow \quad b^\dagger_\ell b_\ell, b^\dagger_{\alpha\ell}, b_{\alpha\ell}, \ell = p, \ldots, N$ do not appear in $\mathbb{L}$

$\Longrightarrow \quad$ commutant $\mathcal{C}$ generated by orthogonal projections

$$P_{\mathbf{n}} = \prod_{k=p}^{N} P_k^{(n_k)} \ , \quad P_k^{(n_k)} = \begin{cases} b_k^\dagger \, b_k & \cdots \quad n_k = 0 \\ b_k \, b_k^\dagger & \cdots \quad n_k = 1 \end{cases}$$

Result 2

Coupling g such that only frequencies $\omega_1 \geq \omega_2 \geq \ldots \omega_{p-1} \geq 0$

$\Longrightarrow$ $b_\ell^\dagger b_\ell, b_{\alpha\ell}^\dagger, b_{\alpha\ell}, \ell = p, \ldots, N$ do not appear in $\mathbb{L}$

$\Longrightarrow$ commutant $\mathcal{C}$ generated by orthogonal projections

$$P_{\mathbf{n}} = \prod_{k=p}^{N} P_k^{(n_k)} \,, \quad P_k^{(n_k)} = \begin{cases} b_k^\dagger\, b_k & \cdots \quad n_k = 0 \\ b_k\, b_k^\dagger & \cdots \quad n_k = 1 \end{cases}$$

Invariant states

$$\rho_{\mathbf{n}} \quad = \quad \frac{P_{\mathbf{n}}\, \rho_\infty\, P_{\mathbf{n}}}{\mathrm{Tr}\Big(\rho_\infty\, P_{\mathbf{n}}\Big)} = \prod_{\ell=1}^{p-1} \Big(\mu_\ell + \nu_\ell\, P_\ell^{(0)}\Big) \prod_{k=p}^{N} \Big(\mu_k + \nu_k\, n_k\Big)\, P_k^{(n_k)}$$

Conclusions and outlook

Conclusions and outlook

- Quantum XX chain of length N with open boundary conditions:
  1. transitions from positive to negative frequencies by varying g

Conclusions and outlook

- Quantum XX chain of length N with open boundary conditions:
  1. transitions from positive to negative frequencies by varying g
- End spins coupled to Bosonic thermal baths at different temperatures without co-rotating terms:
  1. Global approach: master equation quadratic in fermionic operators
  2. Independent dynamics of each Fermionic degree of freedom

Conclusions and outlook

- Quantum XX chain of length N with open boundary conditions:
  1. transitions from positive to negative frequencies by varying g
- End spins coupled to Bosonic thermal baths at different temperatures without co-rotating terms:
  1. Global approach: master equation quadratic in fermionic operators
  2. Independent dynamics of each Fermionic degree of freedom
- Asymptotic states:
  1. Discontinuity in stationary states with respect to g
  2. Expected discontinuities in transport properties
  3. discrimination of global vs local approach

Conclusions and outlook

- Quantum XX chain of length N with open boundary conditions:
  1. transitions from positive to negative frequencies by varying g
- End spins coupled to Bosonic thermal baths at different temperatures without co-rotating terms:
  1. Global approach: master equation quadratic in fermionic operators
  2. Independent dynamics of each Fermionic degree of freedom
- Asymptotic states:
  1. Discontinuity in stationary states with respect to g
  2. Expected discontinuities in transport properties
  3. discrimination of global vs local approach
- Which consequences of phase transitions on the generator $\mathbb{L}$?

  at $g = \dfrac{\Delta}{2}$ the gap of $H_N(g)$ closes