

Two-Day Workshop to celebrate the 60th anniversary of the
paper on dynamical maps by
E. C. G. Sudarshan, P. M. Mathews and J. Rau
October 14 -15, 2021



NITheCS
National Institute for
Theoretical and
Computational Sciences

Contents

Details of the Workshop	2
Organising committee	2
Programme	3
Abstracts of talks	5
Saverio Pascazio	5
Fabio Benatti	5
Adrián A. Budini	6
Daniel Burgarth	6
Dariusz Chruściński	7
Gen Kimura	7
Sergey N. Filippov	8
Arul Lakshminarayan	8
Sabrina Maniscalco	9
Jyrki Piilo	10
A. R. P. Rau	10

Details of the Workshop

The most general dynamical equation of an open quantum system was addressed in a remarkable paper [Phys. Rev. 121, 920 \(1961\)](#) by Sudarshan, Mathews, and Rau (SMR) entitled “Stochastic Dynamics of Quantum Mechanical Systems”.

The paper introduced the notion of a dynamical matrix, associated with any quantum map. Positivity of this Hermitian matrix implies that the corresponding map is completely positive. However, the paper did not receive enough attention from the community, as the notion of complete positivity was not yet introduced.

The workshop organized by [NitheCS](#) and Institute of Theoretical Physics, Jagiellonian University ([IFT UJ](#)) aims to celebrate the 60th anniversary of this fundamental paper and is devoted to new frontiers in the research in open quantum systems and entanglement.

Organising committee

VINAYAK JAGADISH
Faculty of Physics, Astronomy
and Applied Computer Science
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FRANCESCO PETRUCCIONE
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Programme

Thursday, 14 October 2021
(Central European Time: GMT+2)

- 15.00-15.05 Karol Życzkowski (Kraków, Poland)
Opening Address
- 15.05-15.40 Guests of Honor
P. M. Mathews, Jayaseetha Rau (Authors of the celebrated paper)
G. Bhamathi, Ashok Sudarshan (Family of E. C. G. Sudarshan)
- 15.40-16.10 Saverio Pascazio (Bari, Italy)
The dynamics of quantum mechanical systems
- 16.15-16.45 Arul Lakshminarayan (Chennai, India)
*Dual and multi-unitary operators, from entanglement to many-body physics,
as a legacy of the SMR paper*
- 16.50-17.10 —COFFEE BREAK—
- 17.10-17.40 Adrián A. Budini (Bariloche, Argentina)
*Memory effects and bidirectional information flows: an approach from Past-Future
Correlations*
- 17.45-18.15 Fabio Benatti (Trieste, Italy)
Open spin chain asymptotic states
- 18.20-18.50 A. R. P. Rau (Baton Rouge, USA)
Geometries and Constructions of Dynamical Maps including Dissipation

Friday, 15 October 2021
(Central European Time: GMT+2)

- 11.00-11.30 Daniel Burgarth (New South Wales, Australia)
The Quantum Zeno Effect and its applications in Quantum Control
- 11.35-12.05 Dariusz Chruściński (Toruń, Poland)
*Universal constraint for relaxation rates of quantum dynamical semigroups I:
A physical manifestation of complete positivity*
- 12.10-12.40 Gen Kimura (Saitama, Japan)
*Universal constraint for relaxation rates of quantum dynamical semigroups II:
Based on r -function approach*
- 12.45-14.00 —LUNCH BREAK—
- 14.00-14.30 Sabrina Maniscalco (Helsinki, Finland)
*Learning to measure: A new adaptive approach to extract information in quantum
algorithms for near-term quantum computers*
- 14.35-15.05 Jyrki Piilo (Turku, Finland)
Quantum jumps and rate operators in open quantum system dynamics
- 15.10-15.40 Sergey N. Filippov (Moscow, Russia)
*Trace decreasing dynamical maps: Entanglement dynamics and quantum
communication rate*
- 15.45-16.00 Francesco Petruccione (Durban, South Africa)
Closing Remarks

Abstracts of talks

Saverio Pascazio

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THE DYNAMICS OF QUANTUM MECHANICAL SYSTEMS

Sudarshan, Mathews and Rau formulated, in an interesting article published in the Physical Review in 1961, the “most general dynamical law for a quantum mechanical system with a finite number of levels”. In modern language, their equation is known as the Kraus representation of a quantum channel. However, Kraus published his equation in 1971, by making use of a crucial ingredient, complete positivity, that was unknown to physicists in the 1960’s. How could Sudarshan, Mathews and Rau derive the Kraus representation without making use of complete positivity? We discuss this issue in historical and physical terms.

Fabio Benatti

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OPEN SPIN CHAIN ASYMPTOTIC STATES

The master equation for a spin chain of length N with end spins attached to two independent thermal baths is derived in the weak-coupling limit by taking into account the global spin Hamiltonian. The manifold of asymptotic states is then studied with and without counter-rotating terms in the spin-bath coupling.

Adrián A. Budini

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MEMORY EFFECTS AND BIDIRECTIONAL INFORMATION FLOWS: AN APPROACH FROM PAST-FUTURE CORRELATIONS

Open quantum systems are intrinsically coupled with their environments. Depending on the underlying parameters, the corresponding quantum dynamical maps may be characterized or not by memory effects. This last ingredient does not necessarily imply that the environment is affected during evolution. Discerning between all these possibilities has been not a trivial task. Here, using an operational approach based on a set of successive measurement processes performed over the system of interest, it is shown that a consistent definition of memory effects [PRL 121, 240401 (2018)] and bidirectional system-environment information flows [PRA 103, 012221 (2021)] can be established.

Daniel Burgarth

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THE QUANTUM ZENO EFFECT AND ITS APPLICATIONS IN QUANTUM CONTROL

The Quantum Zeno Effect (described in another pioneering article by Misra and Sudarshan in 1977) is amongst the most striking features of Quantum Theory. Frequent observation of a quantum system can completely freeze its dynamics. Therefore in the quantum world, the saying “a watched pot never boils” is really true! Besides its philosophical significance, the Quantum Zeno Effect can also be used to protect a quantum system from noise, and more generally to design and control effective interactions. This subject, Quantum Zeno Dynamics, has recently attracted considerable theoretical and experimental interest, partly fueled by Quantum Computation. In this talk I will show that Zeno dynamics can be induced by dynamical maps and highlight an experimental implementation using superconducting qubits.

Dariusz Chruściński

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UNIVERSAL CONSTRAINT FOR RELAXATION RATES OF QUANTUM DYNAMICAL SEMIGROUPS I: A PHYSICAL MANIFESTATION OF COMPLETE POSITIVITY

We find a new constraint for relaxation rates that holds in fairly large classes of quantum dynamics. It is conjectured that this constraint is universal, i.e., it is valid for all quantum dynamical semigroups. The conjecture is supported by numerical analysis. Since relaxation rates are directly measured in the experiment the presented conjecture provides a physical manifestation of complete positivity. Possible implication of this constraints are also discussed.

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UNIVERSAL CONSTRAINT FOR RELAXATION RATES OF QUANTUM DYNAMICAL SEMIGROUPS II: BASED ON r -FUNCTION APPROACH

In order to find universal constraints for the relaxation rates (spectra) of GKLS master equations, which reflect the physical manifestation of complete positivity condition, we introduce an r -function with which relaxation rates of GKLS generators can nicely be expressed. By investigating an upper bound of the r -function, we derive such universally satisfied constraints tighter than any previously known constraints. A relation with the Böttcher-Wenzel inequality is also discussed.

[1] G. Kimura, Restriction on relaxation times derived from the Lindblad-type master equations for two-level systems, *Phys. Rev. A* 66, 062113 (2002).

[2] G. Kimura, S. Ajisaka, K. Watanabe, Universal constraints on relaxation times for d-level GKLS master equations, *Open Syst. Inf. Dyn.* 24, 1740009 (2017).

[3] D. Chruściński, G. Kimura, A. Kossakowski, Y. Shishido, Universal constraint for relaxation rates for quantum dynamical semigroup, *Phys. Rev. Lett.* 127 (2021) 050401.

[4] D. Chruściński, R. Fujii, G. Kimura, H. Ohno, arXiv:2106.08016 (to appear in LAA).

Sergey N. Filippov

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**TRACE DECREASING DYNAMICAL MAPS: ENTANGLEMENT DYNAMICS AND
QUANTUM COMMUNICATION RATE**

Trace decreasing quantum operations naturally emerge in experiments involving postselection. However, the experiments usually focus on dynamics of the conditional output states as if the dynamics were trace preserving. Here we show that this approach leads to incorrect conclusions about the dynamics divisibility, namely, one can observe an increase in the trace distance or the system-ancilla entanglement although the trace decreasing dynamics is completely positive divisible. We propose solutions to that problem and introduce proper indicators of the information backflow and the indivisibility. We also review a recently introduced concept of the generalized erasure dynamics that includes more experimental data in the dynamics description [1]. We present entangled encodings for quantum information transmission, for which the superadditivity of quantum communication rate takes place. The ideas are illustrated by explicit physical examples of polarization dependent losses.

[1] S. N. Filippov. Capacity of trace decreasing quantum operations and superadditivity of coherent information for a generalized erasure channel. *J. Phys. A: Math. Theor.* 54, 255301 (2021).

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**DUAL AND MULTI-UNITARY OPERATORS, FROM ENTANGLEMENT TO
MANY-BODY PHYSICS, AS A LEGACY OF THE SMR PAPER**

In 1961, Sudarshan, Mathews and Rau defined a “Dynamical matrix” related to the allowed dynamical transformation of density matrices. It turns out that this “realignment” or “reshuffling” operation plays central roles in the study of entanglement of states and operators. This talk presents examples of this from (i) the resolution of a long standing problem of the existence of some absolutely maximally entangled states and the (ii) definition of maximally entangled unitary operators used in quantum circuits as models of many-body nonintegrable systems.

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LEARNING TO MEASURE: A NEW ADAPTIVE APPROACH TO EXTRACT INFORMATION IN QUANTUM ALGORITHMS FOR NEAR-TERM QUANTUM COMPUTERS

Just like their classical counterparts, quantum algorithms require a set of inputs, provided for example as real numbers, and a list of operations to be performed on some reference initial state. Unlike classical computers, however, information is stored in a quantum processor in the form of a wavefunction, thus requiring special procedures to read out the final results. In fact, it is in general neither possible nor convenient to fully reconstruct this quantum state, so that useful insights must be extracted by performing specific observations.

Unfortunately, the number of measurements required for many popular applications is known to grow unsustainably large with the size of the system, even when only partial information is needed. This is for example the case for the so-called Variational Quantum Eigensolver, which is based on the reconstruction of average energies. In this talk I will discuss a novel scheme to tackle this problem. We employ a generalised class of quantum measurements that can be iteratively adapted to minimize the number of times the target quantum state should be prepared and observed. As the algorithm proceeds, it reuses previous measurement outcomes to adjust its own settings and increase the accuracy of subsequent runs. We make the most out of every sample by combining all data produced while fine-tuning the measurement into a single, highly accurate estimate of the energy, thus decreasing the expected runtime by several orders of magnitude. Furthermore, all the measurement data contain complete information about the state: once collected, they can be reused again and again to calculate any other property of the system without additional costs.

Jyrki Piilo

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QUANTUM JUMPS AND RATE OPERATORS IN OPEN QUANTUM SYSTEM DYNAMICS

Monte Carlo methods with quantum jumps are widely used to unravel and solve open system master equations [1]. For Lindblad-Gorini-Kossakowski-Sudarshan master equation and semigroup dynamics, Monte Carlo wavefunction (MCWF) method was developed almost 30 years ago [2]. It was generalized to open system dynamics with memory effects with non-Markovian quantum jump (NMQJ) method in 2008 [3]. However, there exists in-between region between the two - also when describing memory effects - when master equations contain negative decay rates but the corresponding dynamical maps are P-divisible. We have developed a framework unifying all the three regimes and also discuss what this implies for the existence - or non-existence - of the measurement scheme interpretations for the unravellings of open system master equations [4]. Time allowing, we also discuss about the inherent freedom to assign the terms to the deterministic and jump parts of the stochastic realizations and how this leads to a large number of qualitatively different unravellings.

[1] M. B. Plenio and P. L. Knight, *Rev. Mod. Phys.* 70, 101 (1998).

[2] J. Dalibard, Y. Castin, and K. Molmer, *Phys. Rev. Lett.* 68, 580 (1992).

[3] J. Piilo, S. Maniscalco, K. Harkonen, and K.-A. Suominen, *Phys. Rev. Lett.* 100, 180402 (2008).

[4] A. Smirne, M. Caiaffa, and J. Piilo, *Phys. Rev. Lett.* 124, 190402 (2020).

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GEOMETRIES AND CONSTRUCTIONS OF DYNAMICAL MAPS INCLUDING DISSIPATION

The 1961 paper that is being commemorated in this workshop, earlier work on density matrices in the 1950s, and further extensions in the 1970s by E. C. G. Sudarshan and others to handle dissipation and decoherence are central to the field of quantum information today. Geometries and symmetries of transformations have played a central role throughout. Compact procedures for constructing the time-evolution operators and their associated geometrical renderings will be discussed.