Introduction to Elementary Particle Physics

3: An (continued) overview of Calculations

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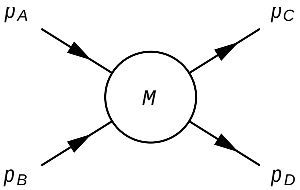
Homework

If the LHC has collected $25\,fb\ ^{1}$ of data, how many proton-proton collisions have they produced? ($_{pp}$ ' $100mb$)

The Second Golden Rule

The *Second Golden Rule*[1] (or Born approximation) is:

$$\underbrace{dW_{fi}}_{\text{interaction rate}} = 2 \underbrace{\int d^3r \; {}_{r} V(r) \; {}_{i}}_{\text{amplitude } (M)}{}^{2} \quad \underbrace{}_{\text{phase space}}$$



$M \quad \langle f|V|i\rangle$ contains the *dynamical* information of the interaction from state $|i\rangle$ to state $|f\rangle$ (potential (V), charge, spin, etc)

contains the *kinematic* information of the interaction $(p_A; p_B; p_C; p_D; \ldots)$

[1] the derivation is beyond the scope of this course

With the Born approximation, an assumption that we work with *spin-averaged* amplitudes, and a few pages of maths, the decay rate for a two-body decay $(A \; ! \; B + C)$ can be shown to be:

$$= \frac{|\mathbf{p}|}{8 \; m_A^2} |M|^2$$

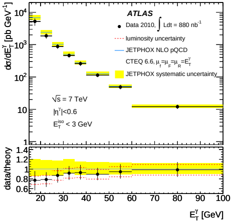
Similarly, the differential cross section for a $2 \; ! \; 2$ scatter $(A + B \; ! \; C + D)$ can be shown to be:

$$\frac{d}{d} = \quad \frac{1}{8} \quad ^2 \frac{|M|^2}{(E_A + E_B)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}$$

Comparing Theory and Experiment

$\mathcal{M}$ can be calculated with the *Feynman Rules*[2], so decay rates and cross sections can be calculated then compared to experiment.

$$\underbrace{\left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_A+E_B)^2}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}}_{\text{theorist calculates}} = \frac{d\sigma}{d\Omega} = \underbrace{\frac{dN_{\text{scat}}}{d\Omega \int L\, dt}}_{\text{experimentalist measures}}$$

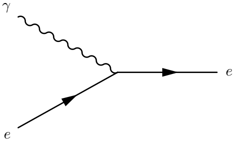


- yellow band is theory calculation
- black points are experimental data

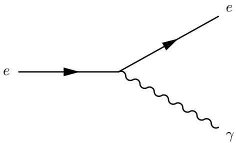
[2] take honours particle physics if you want to see how

Particle Interactions

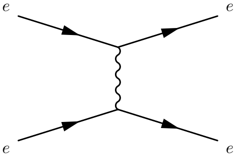
Force is transmitted when a fermion emits or absorbs a boson:



electron absorbs photon



electron emits photon



electrons exchange photon

$\overrightarrow{time}$

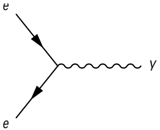
These are called *Feynman diagrams*.

- time flows left to right
- arrow denotes particle (forward) or antiparticle (backward)
- the vertical axis has no physical meaning

Propagators

The (unobserved) particle exchanged is called the *propagator*.

Look at the first half of the $e^+e \quad ! \; e^+e$ diagram:



Can you conserve 4-momentum (p) here?

Particles that are `o *mass shell'* are called *virtual* particles.

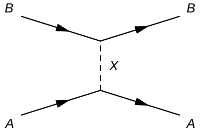
- propagators are virtual
- initial and final state particles are *real*

A particle can be virtual provided doesn't live too long:

$$E \quad t \quad \frac{1}{2}$$

Range of a Force

Take the general $AB \rightarrow AB$ interaction via particle X.



Look at lower vertex in A rest frame ($\mathbf{p}_A^{initial} = 0$)

$$(m_A; 0) \rightarrow (E_A; \mathbf{p}_A) + (E_X; \mathbf{p}_X)$$

So,

$$\begin{aligned} E &= E_f \quad E_i \\ &= E_A + E_X \quad m_A \\ &= \sqrt{\mathbf{p}_A^2 + m_A^2} + \sqrt{\mathbf{p}_A^2 + m_X^2} \quad m_A \end{aligned}$$

The limit case $\mathbf{p}_A \rightarrow 0$ gives $E = m_X$, so $E \quad m_X$

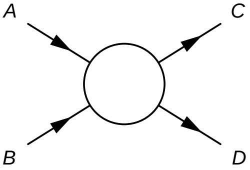
Therefore, Heisenberg says

$$E \quad t \quad \frac{1}{2} \quad \rightarrow \quad t \quad \frac{1}{2 \quad E} \quad \rightarrow \quad t_{unmeasurable} = \frac{1}{2m_X}$$

Massive propagators have limited *range*, R (remember $c = \hbar = 1$).

Mandelstam Variables

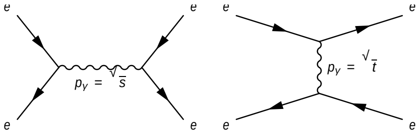
Mandelstam[3] *variables* are Lorentz invariants in 2 ! 2 interactions:



$$s \qquad (p_A + p_B)^2$$
$$t \qquad (p_A \quad p_C)^2$$
$$u \qquad (p_A \quad p_D)^2$$

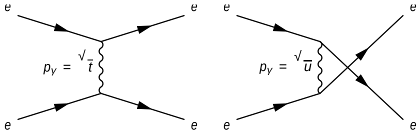
Examples:

$e^+ e \quad ! \quad e^+ e$



s-channel t-channel

$e \quad e \quad ! \quad e \quad e$



t-channel u-channel

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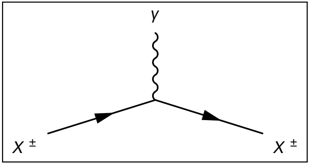
[3] South African, BSc from Wits in 1952

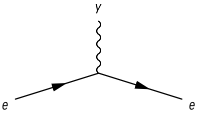
Quantum Electrodynamics (QED)

Electromagnetism mediated by the photon and described by QED.
Every QED interaction is based on this *vertex*:

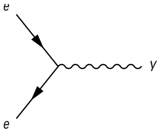


- the solid line (X) is any electromagnetically charged particle
- the squiggly line is a photon ( )
- the *coupling constant* is $= \frac{1}{137}$

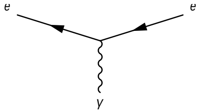
The vertex can be rotated to give other processes:



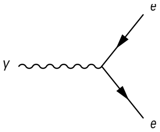
e scatter



$e^+ e$ annihilation

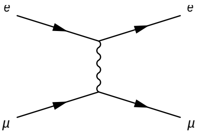


e^+ scatter

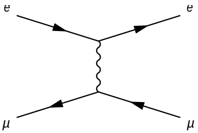


$e^+ e$ pair production

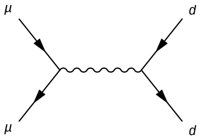
Some Examples of QED Interactions



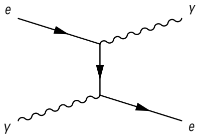
e ! e



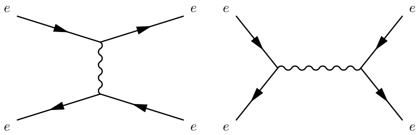
e + ! e +



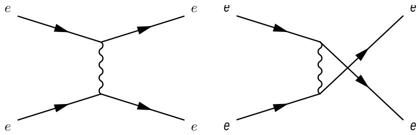
+ ! dd



e ! e



e + e ! e + e

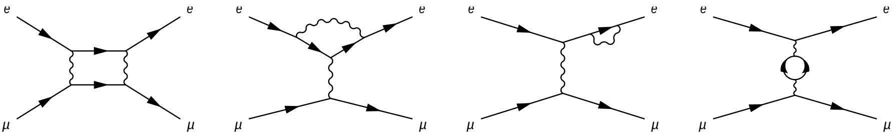


e e ! e e

Higher Order Diagrams

The previous examples are the *lowest order (LO)* diagrams for the processes. Every process has *higher order* diagrams.

Next-to-leading order (NLO) diagrams for *e* ! *e* are:

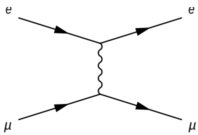


Higher order diagrams are constructed by adding additional *internal lines* without adding *external lines*.

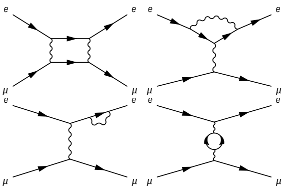
Note that each diagram is constructed of the fundamental QED vertex, each vertex with a 'strength' proportional to .

Perturbation Theory

To calculate what happens in an interaction like $e \quad ! \; e \quad$,
one must add the diagrams at every order:



+



+

$O(\;^2)$ + $O(\;^4)$ +

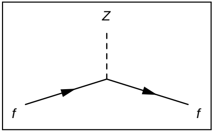
Because $\;< 1$, each higher order contributes a smaller amount to the result. Phew!

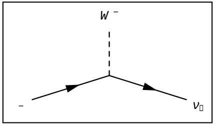
Weak Interactions

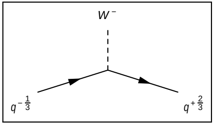
Weak interactions are mediated by W and Z

- the weak charge is rather complex...
- all fermions carry weak charge
- W boson couples charged leptons to neutrinos
- W boson can also change quark flavour

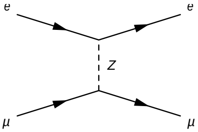
There are 3 weak interaction vertices:



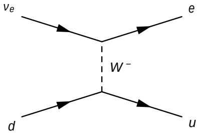




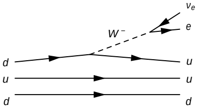
Weak Interaction Examples



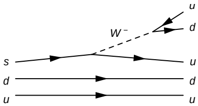
e + ! e +



d + e ! u + e



n ! p + e + e



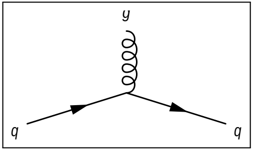
! p +

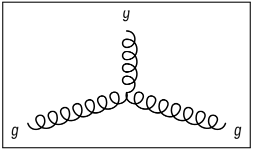
Quantum Chromodynamics (QCD)

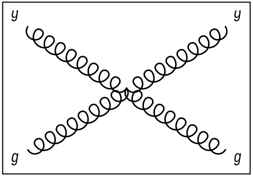
QCD describes the strong interaction mediated by the gluon

- the charge of the strong interaction is *colour*
- colour comes in 3 types: *red, green, blue* (plus anti-colours)
- only quarks and gluons carry colour charge

There are 3 fundamental QCD vertices:







The strong *coupling constant* is $_s$ & 1

Freedom and Confinement

The gluon carries colour, unlike the photon which does not carry electric charge, this has consequences...

Asymptotic Freedom:

- coupling constants: $_s$ & 1, while < 1
- thankfully, at small distances, $_s$ becomes < 1, so perturbation theory can be used for some QCD calculations
- this is called *asymptotic freedom* (quarks are "free to move around" inside a proton)

Confinement:

- no naturally occurring particles carry colour
- quarks are *con ned* to bound states with no net colour charge
- particles composed of quarks are called *hadrons*

Hadron Classification

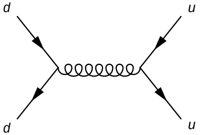
Hadron: a particle made of quarks is called a *hadron*.

- *Meson:* a hadron made of a quark-antiquark pair
- *Baryon:* a hadron made of three quarks or three antiquarks

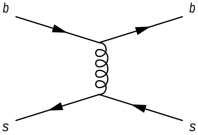
Examples:

	Quark Content	Spin	Charge	Mass (MeV)
Baryon				
p	uud	1=2	+1	938
p	uud	1=2	-1	938
n	udd	1=2	0	939
^{0}	uds	1=2	0	1192
$^+$	uud	3=2	+1	1232
$^{++}$	uuu	3=2	+2	1232
Meson				
^{0}	$(uu\;\; dd)/\sqrt{2}$	0	0	135
	ud, du	0	1	140
	ud, du	1	1	775
K	us, su	0	1	494
D	cd, dc	0	1	1869
B	ub, bu	0	1	5279
	cc	1	0	3097
	bb	1	0	9460

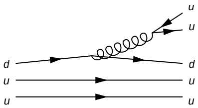
Example Diagrams



dd ! uu



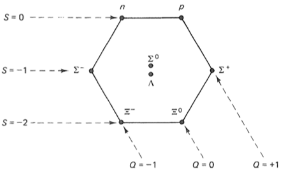
bs ! bs



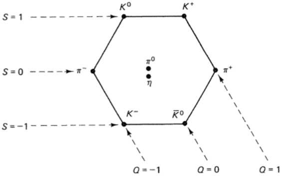
+ ! p + 0

Before the strong interaction was understood:

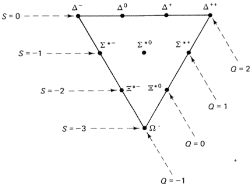
- many ‘fundamental’ particles were observed (m . 2 GeV)
- the particles were arranged in patterns, Gell-Mann called it the “The Eightfold Way”



baryon octet



meson octet

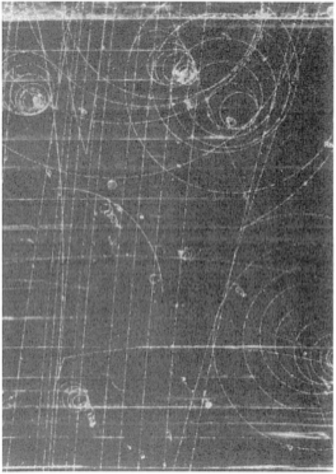


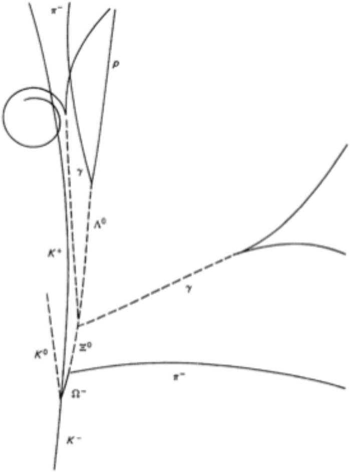
baryon decuplet

The symmetry indicates that hadrons are composite particles.

The Omega Minus

Based on the baryon decuplet, Gell-Mann predicted the



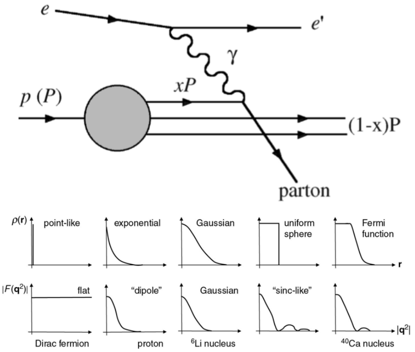


What about the Pauli exclusion principle?

Evidence for Quarks: Lepton-Nucleon Scattering

Similar to Rutherford's discovery of the nucleus

- bombard protons and neutrons with electron 'probes'
- if nucleons are made of *partons* the resulting differential cross section will show the internal structure



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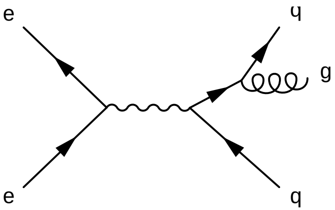
Evidence for Quarks: Jet Production

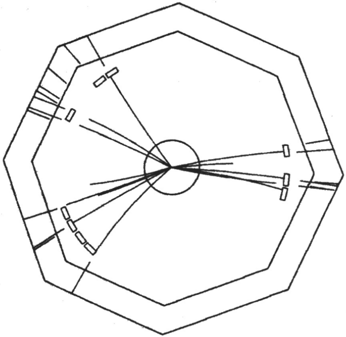
Jets (a columnated flow of hadrons) are observed in electron-positron collisions.

- underlying process $e^+ + e^- \to q + \bar{q}$
- outgoing quarks form hadrons due to confinement, this is called *hadronization*

Gluons

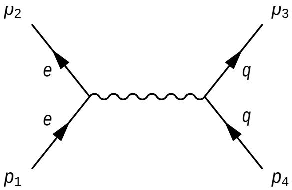
Gluons can also be produced, in e^+e collisions:





Colour Charge

Most direct evidence of colour comes from $R \quad \frac{(ee\ !\ hadrons)}{(ee\ !\ \quad)}$.



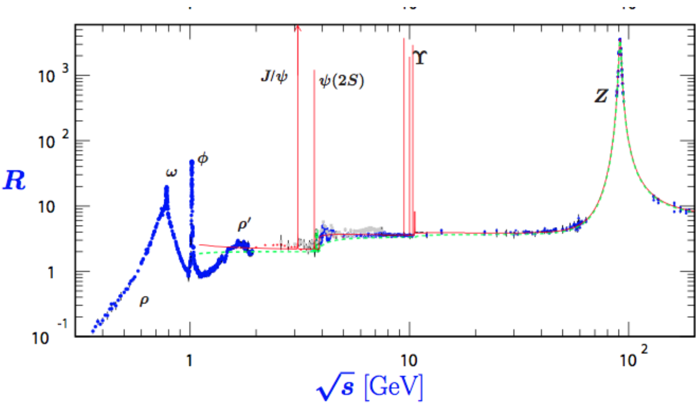
$$! \qquad = \frac{}{3} \quad \frac{Q}{E} \quad ^2$$

where Q is the charge in units of e ($\frac{2}{3}$ for $u; c; t$ and $\frac{1}{3}$ for $d; s; b$)

- if $E < 2m_q$, quark production is kinematically forbidden
- increases when heavier quarks are energetically allowed

If we assume quarks carry 3 colours: $R(E) = 3 \,\mathrm{P}\, Q_i^2$

$$R \rightarrow \underbrace{3\left[\left(\tfrac{2}{3}\right)^2 + 2\left(-\tfrac{1}{3}\right)^2\right]}_{2 \text{ for } E < 2m_c} \rightarrow \underbrace{3\left[2\left(\tfrac{2}{3}\right)^2 + 2\left(-\tfrac{1}{3}\right)^2\right]}_{3{:}33 \text{ for } E < 2m_b} \rightarrow \underbrace{3\left[2\left(\tfrac{2}{3}\right)^2 + 3\left(-\tfrac{1}{3}\right)^2\right]}_{3{:}67 \text{ for } E < 2m_t}$$



R does not describe hadronic resonances, but:

- the factor of 3 is clearly needed to describe data
- strong evidence of quarks carrying 3 colours