A New Horizon for Teaching and Learning of Mathematics

A NITheCS Colloquium Talk by Zurab Janelidze

On Sunday (yesterday), I asked my students to send me some comments about the teaching and learning approach in Foundations of Abstract Mathematics. Two students responded immediately.

Afternoon Prof. Thanks for giving us the platform to learning in a new way. I am personally coming from the chemistry perspective/theory based learning last semester when you called me for an engagement with the solutions I wrote for the set theory test you pointed out a very important point that "You lack the basics of understanding the language of mathematics and your work is so hard to understand without your one-on-one explanation". I went out to my place crying tears I was so heartbroken and didn't want anything to do with mathematics anymore. It was so painful and difficult to understand.

But All I can say now Prof. Thanks for your words they really did unveiled the real person I was meant to be as a student and as a future learned intellectual. I realized that your teaching approach is too personal and it builds one's desire to improve their ability of approach and problem solving in a reasonable way. I can safely say you've opened up a dimension and approach to teaching and learning not only just mathematics including all subjects taught at the tertiary level. Yes this is a new thing to emerge from someone in the academic space and not some political space. Your work is very phenomenal. I put the blame to myself for not putting the effort in THE FOUNDATIONS OF ABSTRACT MATHEMATICS but I have learned from my mistakes and am willing to comeback full on and maybe have sessions with you to see how well I progress with the module.

Your approach to give students enough time to work on their own with the help of tutors who are our peers is very good since this gives us the opportunity to engage actively, specially students who are scared to ask questions during the lecture. On the other hand, confidence is built from seeing how we fumble along the journey. You've mastered the art of Philosophy in Mathematics. Now I no longer view mathematics as a subject but like you said, "a concept". You've taught me amongst other things the urge to "CHALLANGE AND CRITICISE MY WORK AT BEST THAT I CAN". Funny enough I was the class representative for Physical Chemistry last semester and one of the days I approached the Lecturer and advised her to approach the module in a way that enforces teamwork or collaboration so to speak. Without negligence we had tutorials every week and we were divided into groups so we can share out ideas and how we solved the problems we were assigned to each Wednesday. I then went to her office towards the end of the semester and she was impressed with the type of engagement and also the results improved using that method of collaborative learning.

Lastly Prof., I would like to say it's very important for the Faculty of Science to try adopting this way of teaching and not just focusing on producing graduates who lack the skill they were meant to master during their studies. This I would like to call a COLLABORATION AND SKILLED APPROACHED LEARNING. One last thing Prof. I'd like to say thank you for your passion in Mathematics and not just that but of the fact that you know and understand what you're doing, your approach is not just a cram pass and forget approach we're used to, but a method that allows individuals to pay attention to detail and reason with caution.

Many Thanks!!!!!
Mlungisi Blessing Mashazi

Zea de Bruyn

Click below to play audio

Instead of repetitive information, the lecturer gives us information and lets us build on it while learning from each other. I wish other modules were like that too.

Comment by a student in Foundations of Abstract Mathematics, Stellenbosch Univeristy, 2021

Sample assignment

Submit the following in a single pdf file:

Section A. Solutions to one of the tasks (proofs) in the incidence geometry notes.

Section B. A self-made theorem in incidence geometry that can be proved using the axioms, and its proof.

Section C. The list of tasks (proofs) from the same notes that you are comfortable with, along with your draft work where you explored their proofs (do not create any new complete proofs, just include draft proofs you already have in your possession).

Your submission will be marked according to the following rubric:

Ability to compose correct but possibly incomplete proofs - 5 marks (based mainly on Section A)

Ability to compose complete proofs - 5 marks (based mainly on Sections A and B)

Creativity and effort - 5 marks (based mainly on Sections B and C)

Sample test series

There will be multiple opportunities for writing the test. In each opportunity, you will be given a theorem from basic set theory that we have not discussed in the course. Once you complete the test, you have an option either to hand in your completed test or to opt for the next opportunity. Note: you are allowed to bring and use your class and working notes during the test; however, they must be in paper format. You are not allowed to use an electronic device during the test.

In each test, you will need to complete the following three tasks based on the theorem:

Task 1. Illustrate the validity of the theorem on an example.

Task 2. Provide illustrations for and intuitive insight into the theorem. If possible, give an intuitive explanation as to why the theorem should be true and back this explanation with illustrations.

Task 3. Provide a clearly structured formal proof of the theorem.

Marking rubric:

Concretization skill shown across all tasks: 5 marks.

Intuitive understanding shown across all tasks: 5 marks.

Competence in the formal mathematical reasoning shown in Task 3: 5 marks.

Hypothesis:

Teaching and learning of mathematics is more effective when anchored at developing mathematical skills rather than accumulating knowledge. In tests and assignments, the skills should be assessed rather than answers or methods.

In July, in my role as the SAMS president, I co-organised a gathering of various national societies and entities within mathematical sciences. We invited Professor Jonathan Jansen to talk to us.

Maths teaching in SA just doesn’t add up — column by Jonathan Jansen for Herald Live

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I decided to address the elephant in every classroom: Why are so many SA children struggling with mathematics?

First, because mathematics is taught without meaning, as a series of operations and little more.

…

Second, because of the fear of failure in mathematics.

Ours is a risk-averse society and in classrooms we stress not experimentation but the right answers.

You are made to look stupid if you get the answer wrong.

Children sense that education is about the right answer and so they are terrified about looking silly in a class of image-conscious teenagers.

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Maths teaching in SA just doesn’t add up — column by Jonathan Jansen for Herald Live

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Seventh, because of the enthusiasm gap in teaching any subject.

You can solve most of our backlogs in science or maths with teachers who match competence with enthusiasm.

Ever watch a SA teacher or lecturer?

Most of them (not all) look as if they have lost a relative.

You sense a burden on their backs.

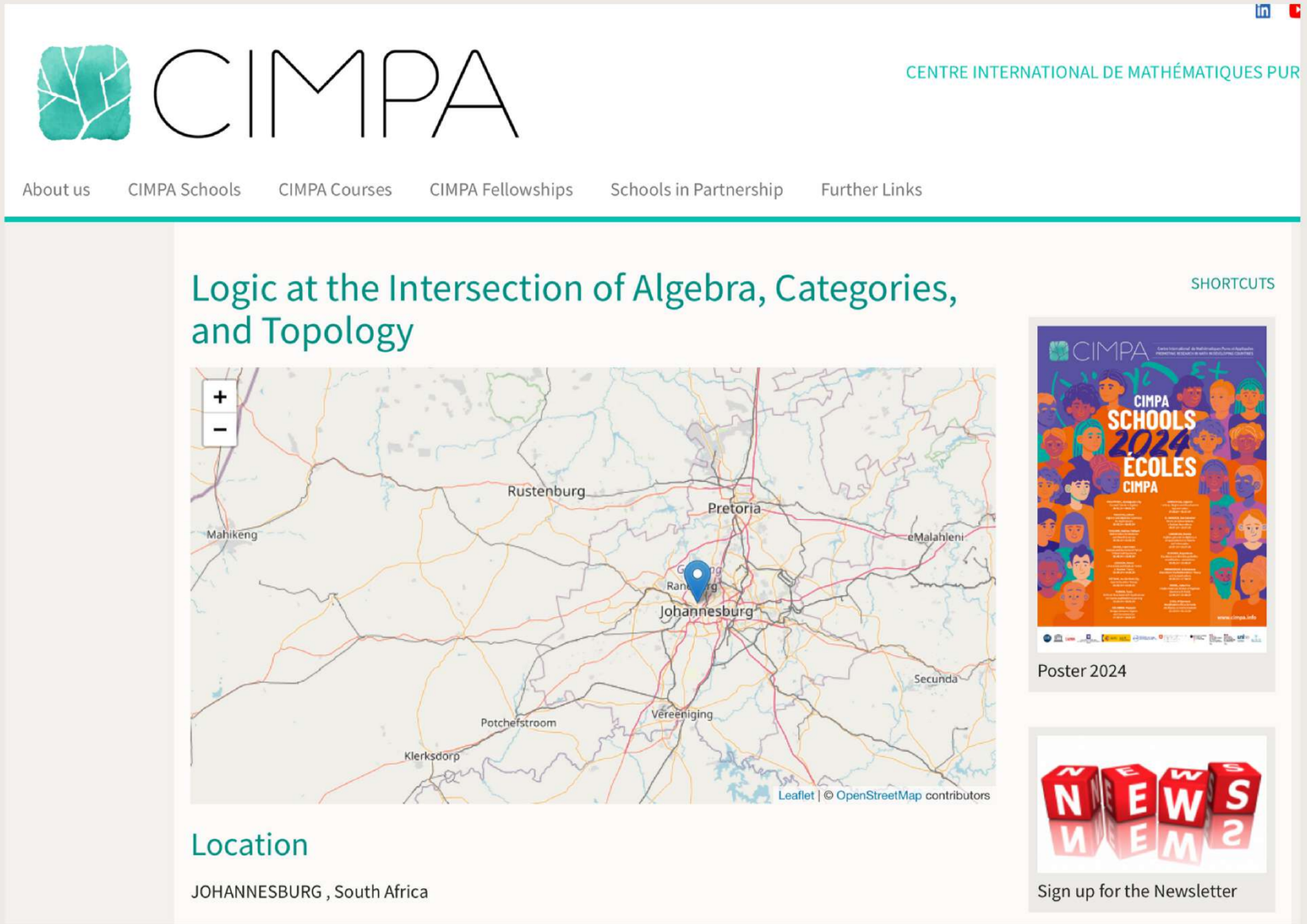
The children observe the misery on their faces and make an early calculation — ‘this is not a job I want one day’.

…

Hypothesis:

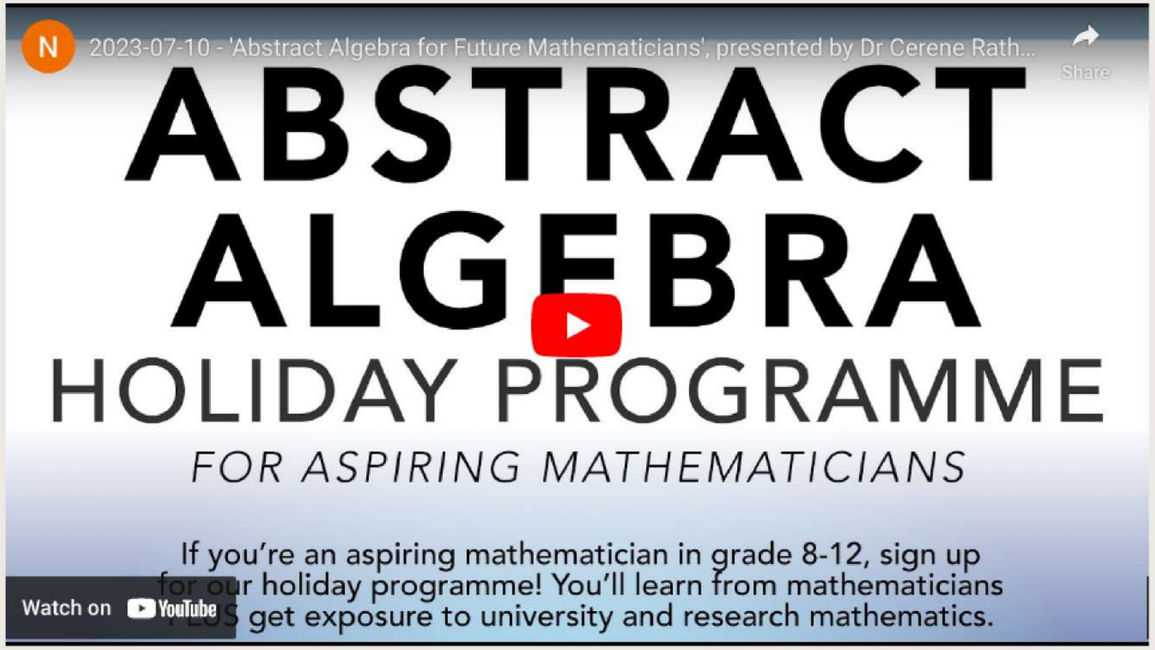
Passion for mathematical insight is the only framework for effective teaching and learning of mathematics.

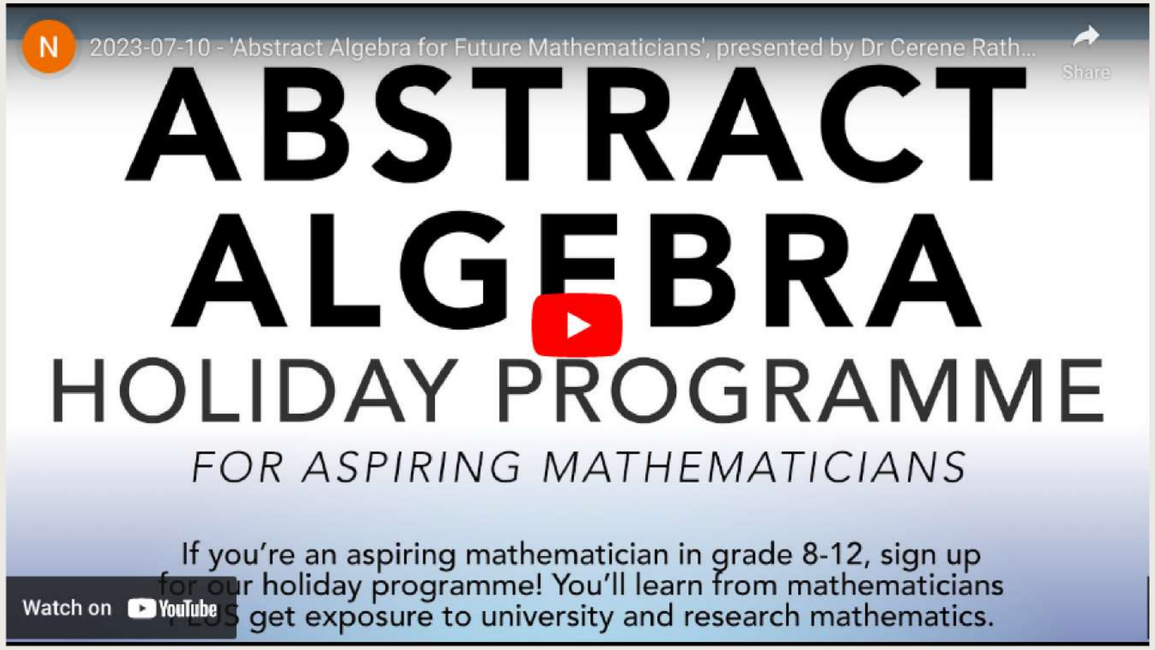
In January, I took up the question of an alternative mathematics programme with participants of a summer school.



At the CIMPA workshop in January 2023, organised with the involvement of Math Structures and Modelling Research Programme at NITheCS, an idea was conceptualised to introduce a meta-curricular (inter)national programme in mathematics that is not based on obtaining marks, but instead is based on cultivating interest and passion.

ABSTRACT ALGEBRA @ NITheCS





"I understood mathematics in just a simplified manner. During this programme the speakers helped me further understand and develop my mathematical skills. The speakers made me feel heard with my questions and helped me further understand my mistakes without belittling my thinking. It gave me a great platform to explore mathematics in a way I understand and enjoy."

THE FUTURE MATHEMATICIAN'S PROGRAMME

The purpose of the Future Mathematicians Programme is to expose school learners and university students to the fundamentals of research mathematics, encourage mathematical dialogue and promote mathematics as a profession.

The programme aims to introduce and accelerate the development of the following skills for school learners and university students:

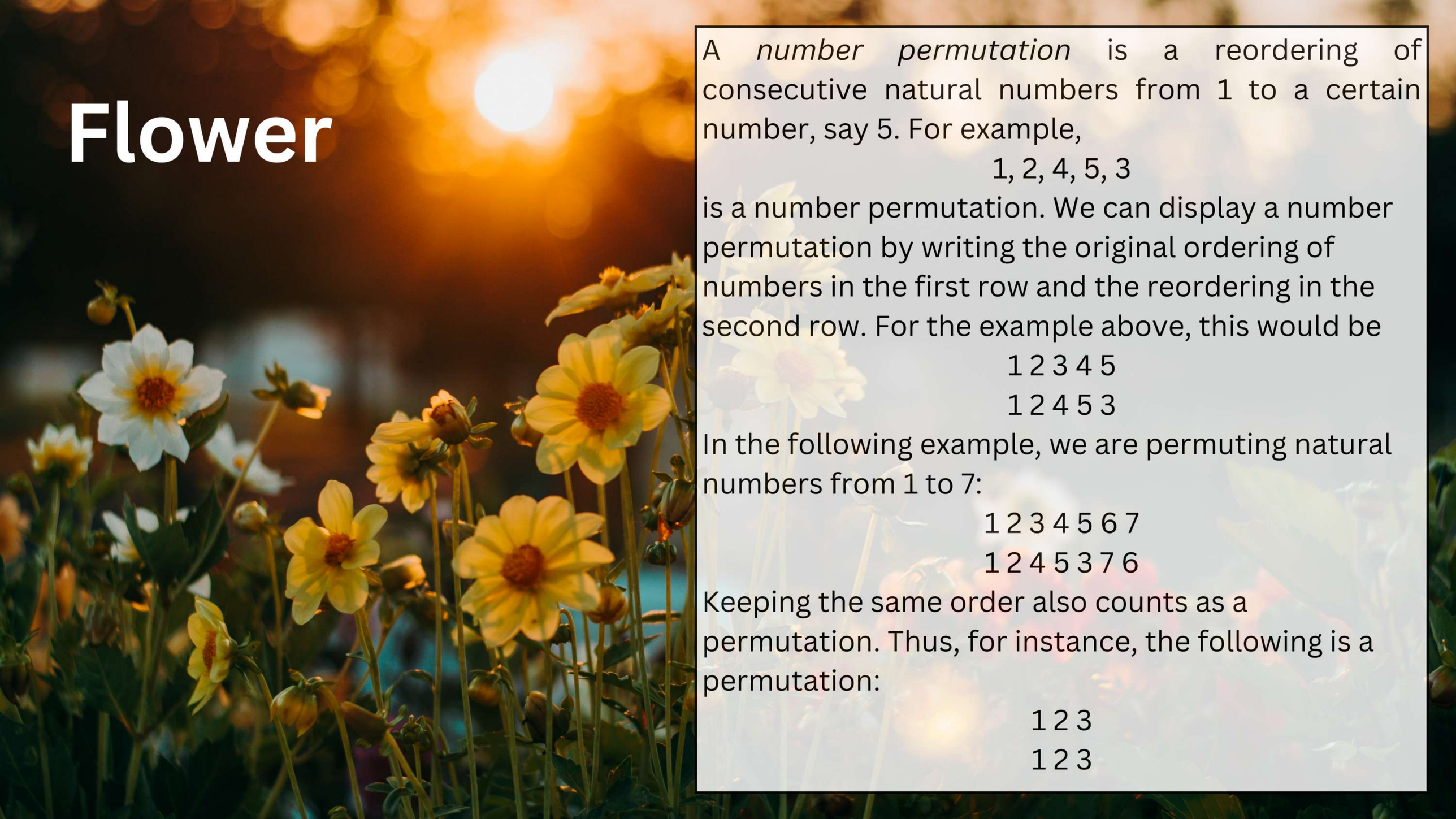
- Use deductive reasoning to make a conclusion and explain it.
- Describe mathematical ideas in an intuitive mathematical language.
- Converse in the language of sets and mathematical structures.
- Use of symbolic language to write precise mathematical statements.
- Abstracting mathematical principles and structures from real life.
- Mathematical problem solving and exploration.
- Comprehending and building a mathematical theory (understanding and creating definitions, axioms, theorems, proofs, and examples).

Hypothesis:

Set theory and logic are not only a solid foundation for formal mathematics, but if correctly approached, can serve as solid foundation for learning mathematical thinking already at the school level.

In 2022, together with collaborators, we have started developing mathematical exploration activities in a mathematics club setting.

Mathematical Exploration Journal



Flower

A *number permutation* is a reordering of consecutive natural numbers from 1 to a certain number, say 5. For example,

1, 2, 4, 5, 3

is a number permutation. We can display a number permutation by writing the original ordering of numbers in the first row and the reordering in the second row. For the example above, this would be

1 2 3 4 5
1 2 4 5 3

In the following example, we are permuting natural numbers from 1 to 7:

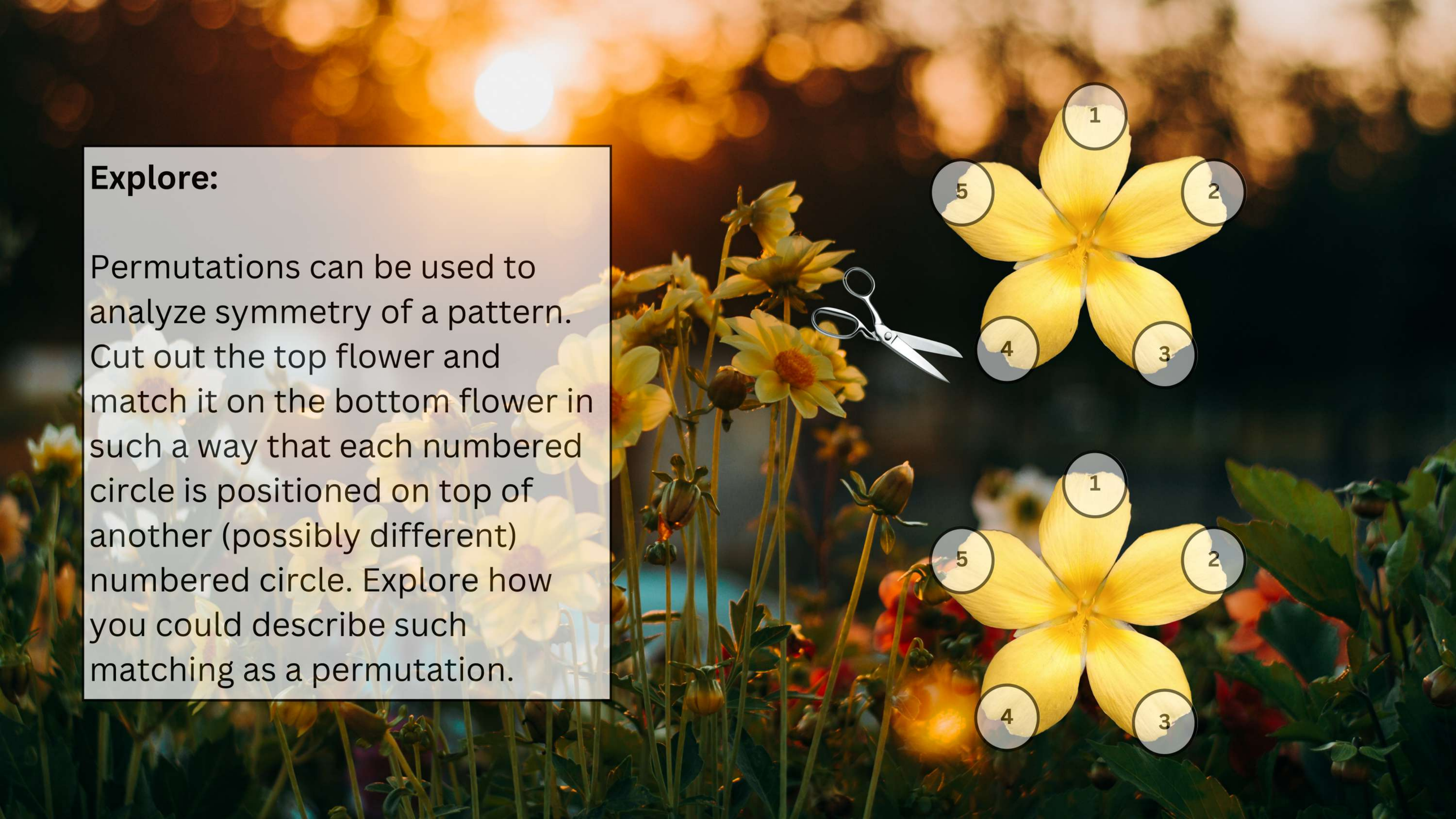
1 2 3 4 5 6 7
1 2 4 5 3 7 6

Keeping the same order also counts as a permutation. Thus, for instance, the following is a permutation:

1 2 3
1 2 3

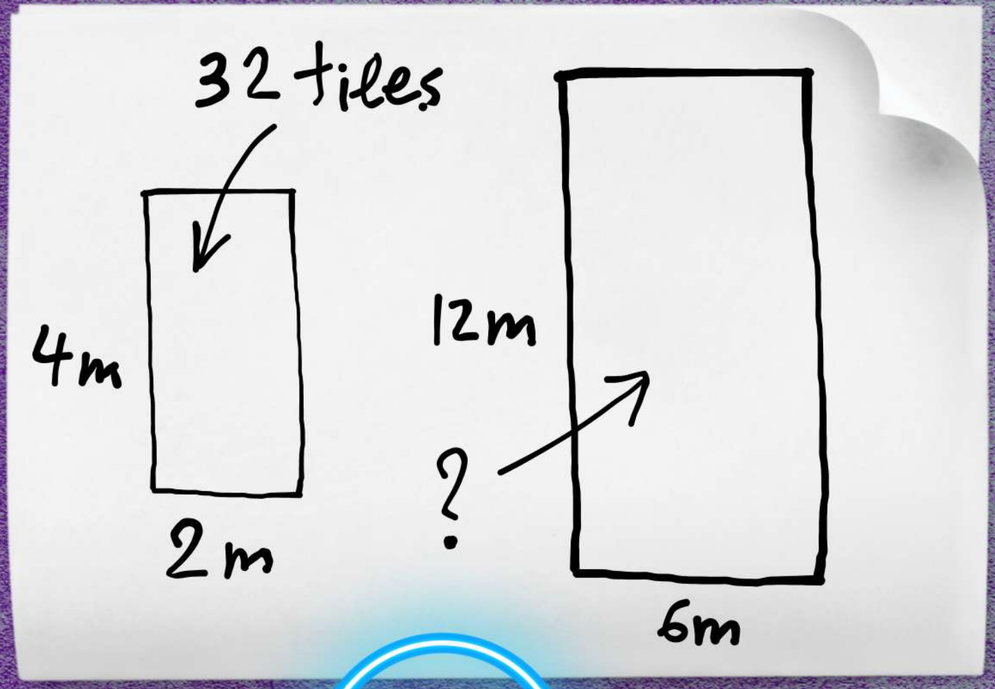
Explore:

Permutations can be used to analyze symmetry of a pattern. Cut out the top flower and match it on the bottom flower in such a way that each numbered circle is positioned on top of another (possibly different) numbered circle. Explore how you could describe such matching as a permutation.



“I had a complete disconnect with mathematics. I viewed mathematics only for intelligent people and failed to see mathematics anywhere but in currency. After our first exploration I was completely convinced that mathematics was everywhere and can be applied by everyone. I started understanding that with logic and real understanding of the concepts, mathematics can be easier for everyone. I learned to ask for help, read to understand not to finish, to apply logic to my work and not box my thinking in what I am taught in school.”

Grade 11 Mathematics Literacy Student



Robot

Assumptions:
The smaller rectangular floor has been tiled. The robot must tile the bigger rectangular floor with tiles of the same kind.

Conclusion:
The robot will need 288 tiles to tile the bigger floor.

Explore:
Why is the conclusion true?

Assumptions:

The astronomer busy is assigning numbers to stars in the constellation diagram of Gemini, where some of the stars are connected by a straight line. She has already assigned four numbers without an error. She uses one of the following rules:

Rule A. Stars that are connected get assigned the same number.

Rule B. Two connected stars get assigned numbers that differ by 2.

Rule C. The number assigned to a star counts the number of stars connected to it.

Conclusion:

When she is done, there will be a pair of connected stars with numbers 4 and 2 assigned to them.

Explore:

Why is the conclusion true?

Hypothesis:

Deductive reasoning is the overlooked component of mathematics education both at the school level and at the university level.

$$x+3 = 5$$

$$x = 2$$

Explain why is x=2 the only solution to the first equation?

NITheCS

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Mathematical Structures and Modelling Research Programme