Introduction to Elementary Particle Physics

3: An (continued) overview of Calculations

Dr. Sahal Yacoob

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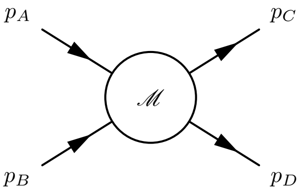
Homework

If the LHC has collected $25\, fb^{-1}$ of data, how many proton-proton collisions have they produced? ($\sigma_{pp} \simeq 100mb$)

The *Second Golden Rule*[1] (or Born approximation) is:

$$\underbrace{dW_r}_{\text{interaction rate}} = 2\pi \underbrace{\left| \int d^3\mathbf{r}\psi_{\mathbf{r}}^{*}\mathbf{V}(\mathbf{r})\psi_{\mathbf{i}} \right|^2}_{\text{amplitude } (\mathscr{M})} \times \underbrace{\rho}_{\text{phase space}}$$



$\mathscr{M} \equiv \langle f|\, V\,|i\rangle$ contains the *dynamical* information of the interaction from state $|i\rangle$ to state $|f\rangle$ (potential (V), charge, spin, etc)

ρ contains the *kinematic* information of the interaction $(p_A, p_B, p_C, p_D, ...)$

[1]the derivation is beyond the scope of this course

Golden Rule for Decay and Scattering

With the Born approximation, an assumption that we work with *spin-averaged* amplitudes, and a few pages of maths, the decay rate for a two-body decay ($A \rightarrow B + C$) can be shown to be:

$$\boxed{\Gamma = \frac{|\mathbf{p}|}{8\pi m_A^2} |\mathscr{M}|^2}$$

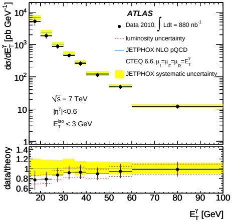
Similarly, the differential cross section for a $2 \rightarrow 2$ scatter ($A + B \rightarrow C + D$) can be shown to be:

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathscr{M}|^2}{(E_A + E_B)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}}$$

Comparing Theory and Experiment

$\mathscr{M}$ can be calculated with the *Feynman Rules*[2], so decay rates and cross sections can be calculated then compared to experiment.

$$\underbrace{\left(\frac{1}{8\pi}\right)^2 \frac{|\mathscr{M}|^2}{(E_A+E_B)^2}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}}_{\text{theorist calculates}} = \frac{d\sigma}{d\Omega} = \underbrace{\frac{dN_{\text{scat}}}{d\Omega\int\mathscr{L}dt}}_{\text{experimentalist measures}}$$

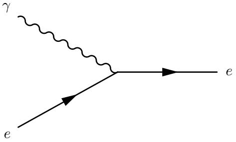


- yellow band is theory calculation
- black points are experimental data

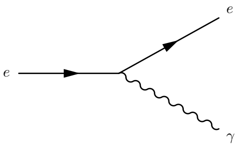
[2] take honours particle physics if you want to see how

Particle Interactions

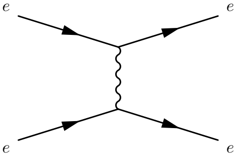
Force is transmitted when a fermion emits or absorbs a boson:



electron absorbs photon



electron emits photon



electrons exchange photon

$\overrightarrow{time}$

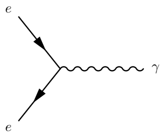
These are called *Feynman diagrams*.

- time flows left to right
- arrow denotes particle (forward) or antiparticle (backward)
- the vertical axis has no physical meaning

Propagators

The (unobserved) particle exchanged is called the *propagator*.

Look at the first half of the $e^+e^- \to e^+e^-$ diagram:



Can you conserve 4-momentum (p) here?

Particles that are *'off mass shell'* are called *virtual* particles.

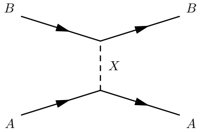
- propagators are virtual
- initial and final state particles are *real*

A particle can be virtual provided doesn't live too long:

$$\Delta E \Delta t \geq \frac{1}{2}$$

Range of a Force

Take the general $AB \to AB$ interaction via particle X.



Look at lower vertex in A rest frame ($\mathbf{p}_A^{initial} = 0$)

$$(m_A, 0) \to (E_A, \mathbf{p}_A) + (E_X, \mathbf{p}_X)$$

So,

$$\begin{aligned}
\Delta E &= E_f - E_i \\
&= E_A + E_X - m_A \\
&= \sqrt{\mathbf{p}_A^2 + m_A^2} + \sqrt{\mathbf{p}_A^2 + m_X^2} - m_A
\end{aligned}$$

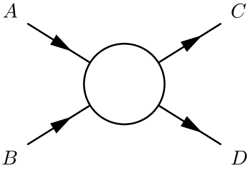
The limit case $\mathbf{p}_A \to 0$ gives $\Delta E = m_X$, so $\Delta E \geq m_X$
Therefore, Heisenberg says

$$\Delta E \Delta t \geq \frac{1}{2} \quad \to \Delta t \geq \frac{1}{2\Delta E} \quad \to \quad \Delta t_{unmeasurable} = \tau \leq \frac{1}{2m_X}$$

Massive propagators have limited *range*, R (remember $c = \hbar = 1$).

Mandelstam Variables

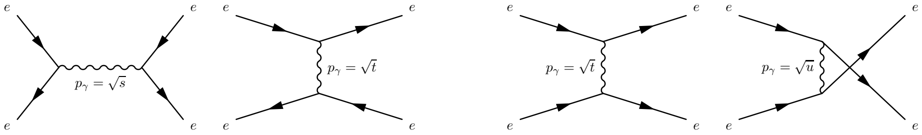
Mandelstam[3] *variables* are Lorentz invariants in $2 \to 2$ interactions:



$$
\begin{aligned}
s &\equiv (p_A + p_B)^2 \\
t &\equiv (p_A - p_C)^2 \\
u &\equiv (p_A - p_D)^2
\end{aligned}
$$

Examples:

$e^+e^- \to e^+e^-$ $\qquad$ $e^-e^- \to e^-e^-$

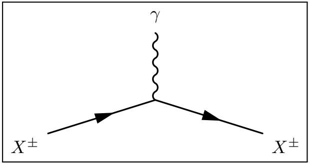


s-channel $\qquad$ t-channel $\qquad$ t-channel $\qquad$ u-channel

[3] South African, BSc from Wits in 1952

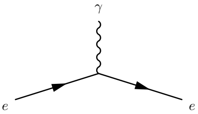
Quantum Electrodynamics (QED)

Electromagnetism mediated by the photon and described by QED.
Every QED interaction is based on this *vertex*:

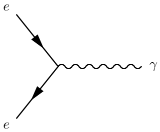


- the solid line ($X^{\pm}$) is any electromagnetically charged particle
- the squiggly line is a photon (γ)
- the *coupling constant* is $\alpha = \frac{1}{137}$

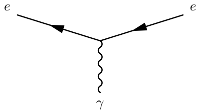
The vertex can be rotated to give other processes:



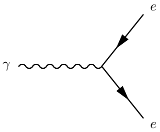
e^- scatter



e^+e^- annihilation

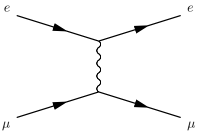


e^+ scatter

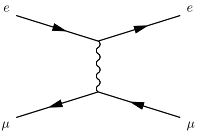


e^+e^- pair production

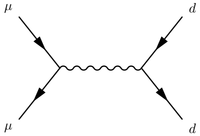
Some Examples of QED Interactions



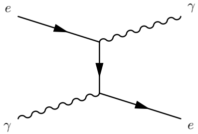
$e^- \mu^- \to e^- \mu^-$



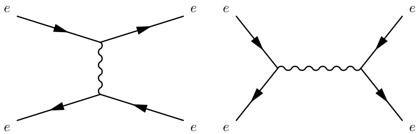
$e^- \mu^+ \to e^- \mu^+$



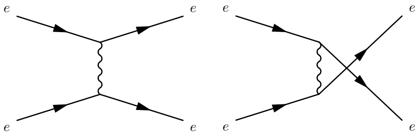
$\mu^+ \mu^- \to d\bar{d}$



$e^- \gamma \to e^- \gamma$



$e^+ e^- \to e^+ e^-$

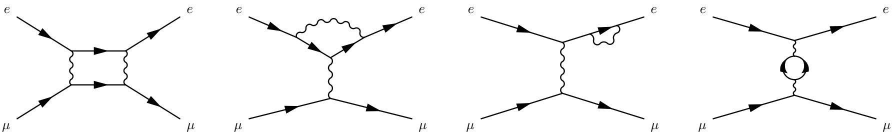


$e^- e^- \to e^- e^-$

Higher Order Diagrams

The previous examples are the *lowest order (LO)* diagrams for the processes. Every process has *higher order* diagrams.

Next-to-leading order (NLO) diagrams for $e^- \mu^- \to e^- \mu^-$ are:

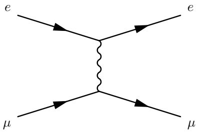


Higher order diagrams are constructed by adding additional *internal lines* without adding *external lines*.

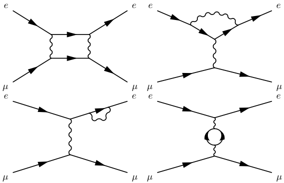
Note that each diagram is constructed of the fundamental QED vertex, each vertex with a 'strength' proportional to α.

Perturbation Theory

To calculate what happens in an interaction like $e^-\mu^- \to e^-\mu^-$, one must add the diagrams at every order:



$+$



$+\cdots$

$\mathcal{O}(\alpha^2) \quad + \quad \mathcal{O}(\alpha^4) \quad + \cdots$

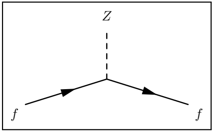
Because $\alpha < 1$, each higher order contributes a smaller amount to the result. Phew!

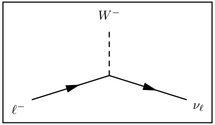
Weak Interactions

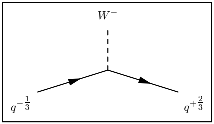
Weak interactions are mediated by W and Z

- the weak charge is rather complex...
- all fermions carry weak charge
- W boson couples charged leptons to neutrinos
- W boson can also change quark flavour

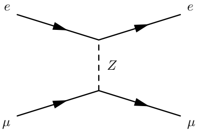
There are 3 weak interaction vertices:



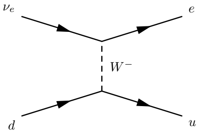




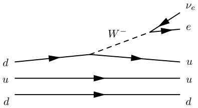
Weak Interaction Examples



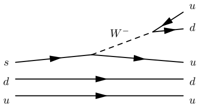
$$e^- + \mu^- \rightarrow e^- + \mu^-$$



$$d + \nu_e \rightarrow u + e^-$$



$$n \rightarrow p + e^- + \bar{\nu_e}$$



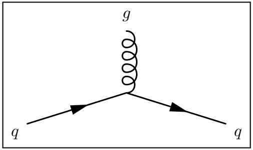
$$\Lambda \rightarrow p + \pi^-$$

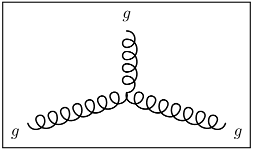
Quantum Chromodynamics (QCD)

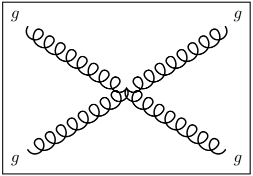
QCD describes the strong interaction mediated by the gluon

- the charge of the strong interaction is *colour*
- colour comes in 3 types: *red, green, blue* (plus anti-colours)
- only quarks and gluons carry colour charge

There are 3 fundamental QCD vertices:







The strong *coupling constant* is $\alpha_s \gtrsim 1$

Freedom and Confinement

The gluon carries colour, unlike the photon which does not carry electric charge, this has consequences...
Asymptotic Freedom:

- coupling constants: $\alpha_s \gtrsim 1$, while $\alpha < 1$
- thankfully, at small distances, α_s becomes < 1, so perturbation theory can be used for some QCD calculations
- this is called *asymptotic freedom* (quarks are "free to move around" inside a proton)

Confinement:

- no naturally occurring particles carry colour
- quarks are *confined* to bound states with no net colour charge
- particles composed of quarks are called *hadrons*

Hadron Classification

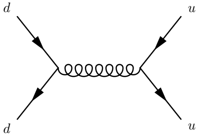
Hadron: a particle made of quarks is called a *hadron*.

- *Meson:* a hadron made of a quark-antiquark pair
- *Baryon:* a hadron made of three quarks or three antiquarks

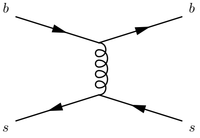
Examples:

	Quark Content	Spin	Charge	Mass (MeV)
Baryon				
p	uud	$1/2$	$+1$	938
$\bar{p}$	$\bar{u}\bar{u}\bar{d}$	$1/2$	-1	938
n	udd	$1/2$	0	939
Σ^0	uds	$1/2$	0	1192
Δ^+	uud	$3/2$	$+1$	1232
Δ^{++}	uuu	$3/2$	$+2$	1232
Meson				
π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	0	0	135
$\pi^\pm$	$u\bar{d}$, $d\bar{u}$	0	± 1	140
$\rho^\pm$	$u\bar{d}$, $d\bar{u}$	1	± 1	775
$K^\pm$	$u\bar{s}$, $s\bar{u}$	0	± 1	494
$D^\pm$	$c\bar{d}$, $d\bar{c}$	0	± 1	1869
$B^\pm$	$u\bar{b}$, $b\bar{u}$	0	± 1	5279
ψ	$c\bar{c}$	1	0	3097
Υ	$b\bar{b}$	1	0	9460

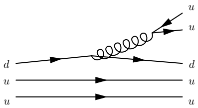
Example Diagrams



$d\bar{d} \to u\bar{u}$



$b\bar{s} \to b\bar{s}$

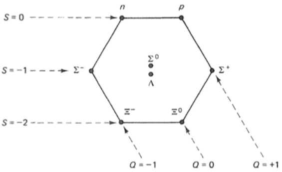


$\Delta^+ \to p + \pi^0$

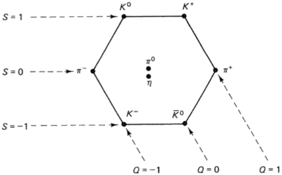
Hadrons and the Strong Interaction

Before the strong interaction was understood:

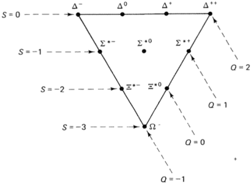
- many 'fundamental' particles were observed ($m \lesssim 2$ GeV)
- the particles were arranged in patterns, Gell-Mann called it the "The Eightfold Way"



baryon octet



meson octet

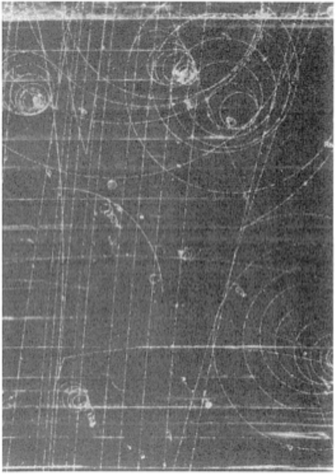


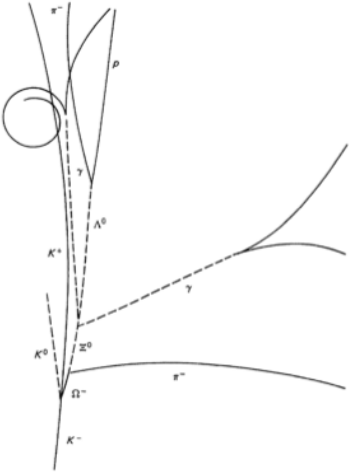
baryon decuplet

The symmetry indicates that hadrons are composite particles.

The Omega Minus

Based on the baryon decuplet, Gell-Mann predicted the Ω^-



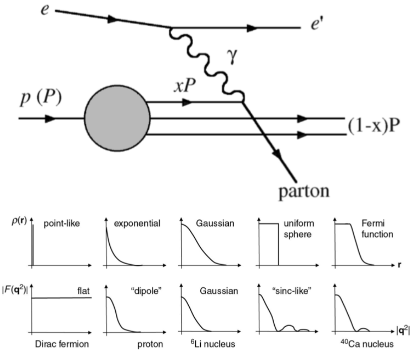


What about the Pauli exclusion principle?

Evidence for Quarks: Lepton-Nucleon Scattering

Similar to Rutherford's discovery of the nucleus

- bombard protons and neutrons with electron 'probes'
- if nucleons are made of *partons* the resulting differential cross section will show the internal structure



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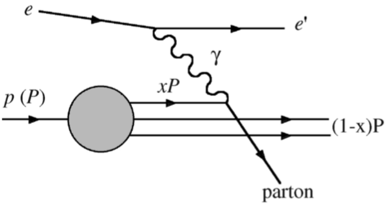

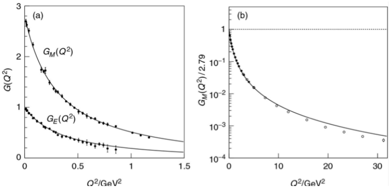

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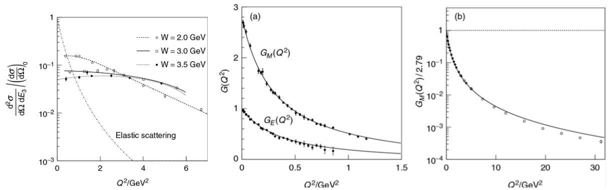
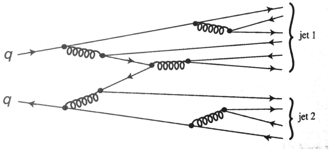
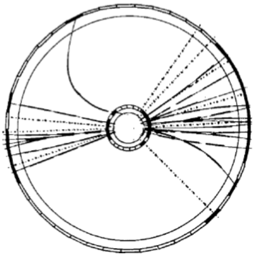


Evidence for Quarks: Jet Production

Jets (a columnated flow of hadrons) are observed in electron-positron collisions.

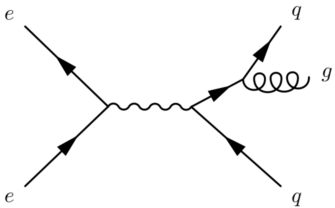
- underlying process $e^+ + e^- \rightarrow q + \bar{q}$
- outgoing quarks form hadrons due to confinement, this is called *hadronization*

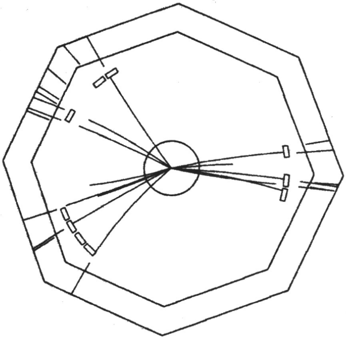




Gluons

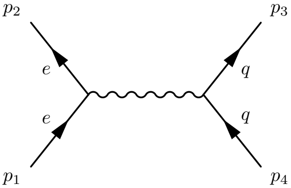
Gluons can also be produced, in e^+e^- collisions:





Colour Charge

Most direct evidence of colour comes from $R \equiv \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)}$.



$\rightarrow$

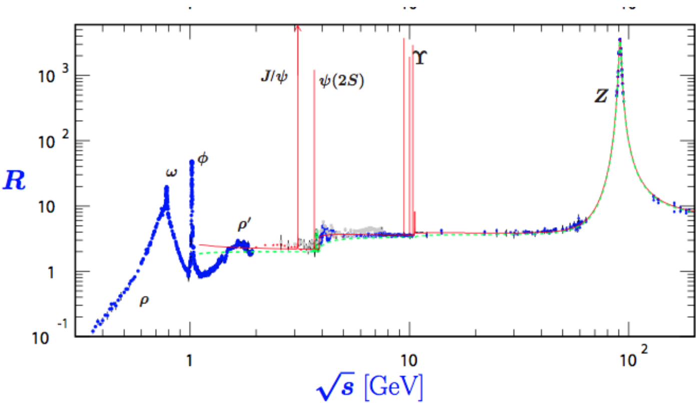
$$\sigma = \frac{\pi}{3}\left(\frac{Q\alpha}{E}\right)^2$$

where Q is the charge in units of e ($\frac{2}{3}$ for u, c, t and $-\frac{1}{3}$ for d, s, b)

- if $E < 2m_q$, quark production is kinematically forbidden
- σ increases when heavier quarks are energetically allowed

If we assume quarks carry 3 colours: $R(E) = 3\sum Q_i^2$

$$R \rightarrow \underbrace{3\left[(\tfrac{2}{3})^2 + 2(-\tfrac{1}{3})^2\right]}_{2 \text{ for } E<2m_c} \rightarrow \underbrace{3\left[2(\tfrac{2}{3})^2 + 2(-\tfrac{1}{3})^2\right]}_{3.33 \text{ for } E<2m_b} \rightarrow \underbrace{3\left[2(\tfrac{2}{3})^2 + 3(-\tfrac{1}{3})^2\right]}_{3.67 \text{ for } E<2m_t}$$



R does not describe hadronic resonances, but:

- the factor of 3 is clearly needed to describe data
- strong evidence of quarks carrying 3 colours