Introduction to Elementary Particle Physics

2: An overview of Calculations

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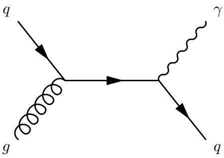
Observable Quantities

2To test a theory, it must predict something measurable.



Theory

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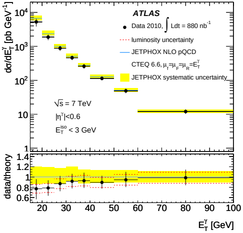

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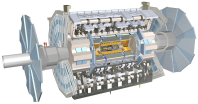


Experiment

The *Second Golden Rule* with the *Feynman Rules* can predict *decay rates* and *cross sections*

- *Decay Rate:* The probability per unit time that a particle will disintegrate.
- *Cross Section:* The 'area' of interaction between particles.

Cross Section

The *cross section* (σ) gives the 'effective area' of an interaction. It is proportional to the probability of the interaction to have a particular outcome.

The dimension of σ is area, commonly used unit is the *barn (b)*[1]:

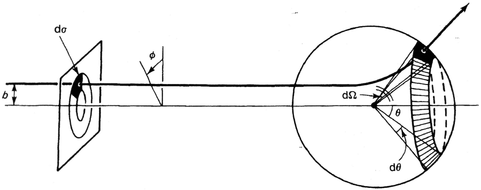
$$1\,b = 10^{-28}\,m^2$$

Note that an individual particle does not have a cross section. The *interaction* between two (or more) particles has a cross section!

[1] comes from the expression "You couldn't hit the broad side of a barn".

Differential Cross Section

In an interaction, particles are *scattered* into different directions.



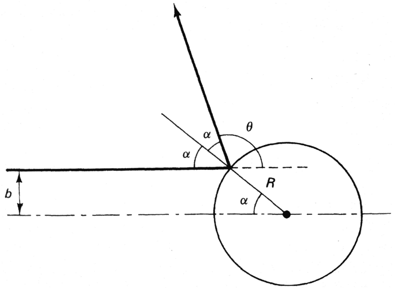
$\frac{d\sigma}{d\Omega}$ is the *differential cross section*, b is called the *impact parameter*.

$$d\sigma = |b\,db\,d\phi| \text{ and } d\Omega = |\sin\theta\,d\theta\,d\phi| \;\rightarrow\; \frac{d\sigma}{d\Omega} = \left|\frac{b}{\sin\theta}\left(\frac{db}{d\theta}\right)\right|$$

$$\frac{d\sigma}{d\Omega} = \frac{\left(\begin{array}{c}\text{Number of interactions per target particle}\\ \text{that lead to scattering into } d\Omega \text{ at angle } \theta\end{array}\right)}{(\text{Number of incident particles } \textit{per unit area})}$$

Cross Section of a Hard Sphere

Differential cross section:



$$\begin{aligned} b &= R\cos(\theta/2) \\ \frac{db}{d\theta} &= -\frac{R}{2}\sin\left(\frac{\theta}{2}\right) \\ \frac{d\sigma}{d\Omega} &= \frac{Rb\sin(\theta/2)}{2\sin\theta} = \frac{R^2}{4} \end{aligned}$$

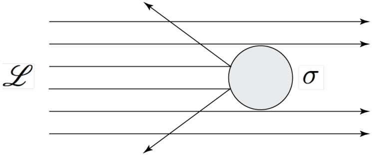
Total cross section:

$$\sigma = \int d\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

$$\sigma = \int \frac{R^2}{4} d\Omega = \pi R^2$$

Luminosity ($\mathscr{L}$) is the number of particles passing (or possible interactions) per unit area per unit time $[cm^{-2}s^{-1}]$.

The number of particles scattered (N_{scat}) per unit time from target of size σ is then:



$$\frac{dN_{\text{scat}}}{dt} = \sigma \mathscr{L}$$

$$N_{\text{scat}} = \sigma \int \mathscr{L}\, dt$$

If the LHC has collected $25\, fb^{-1}$ of data, how many proton-proton collisions have they produced? ($\sigma_{pp} \simeq 100mb$)

Decay Rates

Most particles decay ($\psi \to e^+e^-$ [2] and $\mu^- \to \nu_\mu \, e^- \, \bar{\nu}_e$)

Particles have no memory. The probability of a muon decaying in the next microsecond is independent of when the muon was created. So, the *decay rate* (Γ) is defined by:

$$N(t) = N(0)\, e^{-\Gamma t}$$

Most particles decay in several ways ($\psi \to e^+e^-$ or $\psi \to \mu^+\mu^-$)

The *total* decay rate is the sum of individual decay rates:

$$\Gamma_{\text{tot}} = \sum_{i=1}^{n} \Gamma_i$$

[2] ψ is also called J/ψ

Lifetime and Branching Ratio

The *mean lifetime* (τ), often just called '*lifetime*', is:

$$\tau = \frac{1}{\Gamma_{\text{tot}}}$$

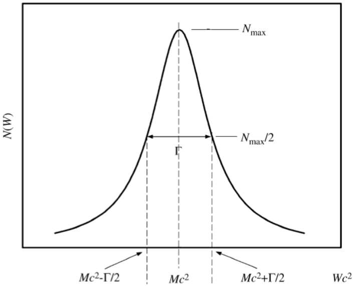
Lifetime is the time in the particle's rest frame.

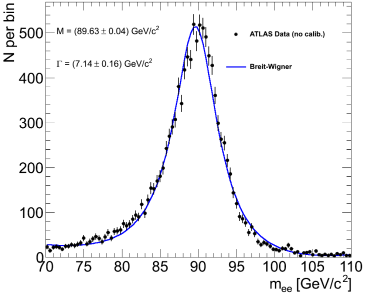
The *branching ratio* is the fraction that decay to a particular mode.

$$\text{Br}(i) = \frac{\Gamma_i}{\Gamma_{\text{tot}}}$$

Unstable particles form *resonances* with a *Breit-Wigner* shape:

$$\sigma \propto \frac{N}{(E-M)^2 + (\Gamma/2)^2}$$

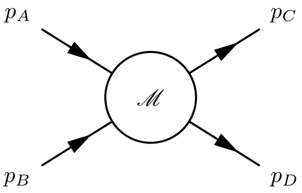




The Second Golden Rule

The *Second Golden Rule*[3] (or Born approximation) is:

$$\underbrace{dW_r}_{\text{interaction rate}} = 2\pi \underbrace{\left| \int d^3\mathbf{r}\psi_{\mathbf{r}}^* \mathbf{V}(\mathbf{r})\psi_{\mathbf{i}} \right|^2}_{\text{amplitude } (\mathscr{M})} \times \underbrace{\rho}_{\text{phase space}}$$



$\mathscr{M} \equiv \langle f | V | i \rangle$ contains the *dynamical* information of the interaction from state $|i\rangle$ to state $|f\rangle$ (potential (V), charge, spin, etc)

ρ contains the *kinematic* information of the interaction $(p_A, p_B, p_C, p_D, ...)$

[3]the derivation is beyond the scope of this course

Golden Rule for Decay and Scattering

With the Born approximation, an assumption that we work with *spin-averaged* amplitudes, and a few pages of maths, the decay rate for a two-body decay ($A \to B + C$) can be shown to be:

$$\boxed{\Gamma = \frac{|\mathbf{p}|}{8\pi m_A^2}|\mathscr{M}|^2}$$

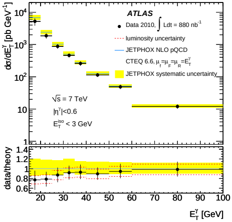
Similarly, the differential cross section for a $2 \to 2$ scatter ($A + B \to C + D$) can be shown to be:

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathscr{M}|^2}{(E_A + E_B)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}}$$

Comparing Theory and Experiment

$\mathscr{M}$ can be calculated with the *Feynman Rules*[4], so decay rates and cross sections can be calculated then compared to experiment.

$$\underbrace{\left(\frac{1}{8\pi}\right)^2 \frac{|\mathscr{M}|^2}{(E_A+E_B)^2}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}}_{\text{theorist calculates}} = \frac{d\sigma}{d\Omega} = \underbrace{\frac{dN_{\text{scat}}}{d\Omega \int \mathscr{L}\, dt}}_{\text{experimentalist measures}}$$



- yellow band is theory calculation
- black points are experimental data

[4] take honours particle physics if you want to see how

From Lagrangian to Particles & Interactions

RECALL...

Classical Mechanics

- define the Lagrangian
  $L = \frac{1}{2}mv^2 - V(x)$
- use the Euler-Lagrange equation
  $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$
- get the equation of motion
  $F = ma$
- you get *Newton's Law*

Quantum Field Theory

- define the Lagrangian (density)
  $\mathcal{L}_f = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$
- use the Euler-Lagrange equation
  $\partial^\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu\psi)}\right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0$
- get the 'equation of motion'
  $(i\gamma^\mu\partial_\mu - m)\psi = 0$
- you get the *Dirac Equation*

The phase of the particle wave function ψ is not observable. Therefore, the Lagrangian should be symmetric under local phase (ie. gauge) transformations. Is it?

Lagrangian Symmetry

The phase (gauge) of a wave function is not an observable, therefore:

- our Lagrangian *should* be symmetric (invariant) under phase transformation, but

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x) \qquad \text{gives} \qquad \mathcal{L}_f \rightarrow \mathcal{L}_f - \bar{\psi}\,\gamma_\mu(\partial^\mu \alpha)\,\psi$$

- it is not symmetric! So what if we add another term to our Lagrangian

$$\mathcal{L}_{f'} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + \underline{e\bar{\psi}\gamma^\mu\psi A_\mu} \quad \text{where} \quad A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu\alpha$$

- we get a symmetric Lagrangian *and* the additional term describes the interaction between a photon and a charged point particle (i.e. an electron)!

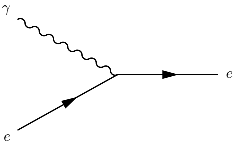
- Then, we can add yet another term to represent the free photon

$$\mathcal{L}_{QED} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi + e\bar{\psi}\gamma_\mu A^\mu\psi - \underline{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

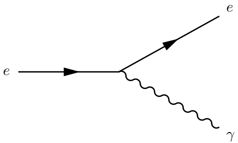
- this is the full Quantum Electrodynamics (QED) Lagrangian

Particle Interactions

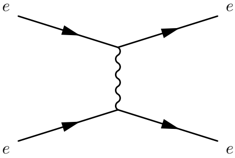
Force is transmitted when a fermion emits or absorbs a boson:



electron absorbs photon



electron emits photon



electrons exchange photon

$\overrightarrow{time}$

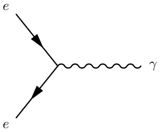
These are called *Feynman diagrams*.

- time flows left to right
- arrow denotes particle (forward) or antiparticle (backward)
- the vertical axis has no physical meaning

Propagators

The (unobserved) particle exchanged is called the *propagator*.

Look at the first half of the $e^+e^- \rightarrow e^+e^-$ diagram:



Can you conserve 4-momentum (p) here?

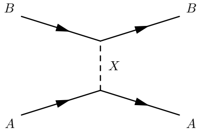
Particles that are *'off mass shell'* are called *virtual* particles.

- propagators are virtual
- initial and final state particles are *real*

A particle can be virtual provided doesn't live too long:

$$\Delta E \Delta t \geq \frac{1}{2}$$

Take the general $AB \to AB$ interaction via particle X.



Look at lower vertex in A rest frame ($\mathbf{p}_A^{initial} = 0$)

$$(m_A, 0) \to (E_A, \mathbf{p}_A) + (E_X, \mathbf{p}_X)$$

So,

$$\begin{aligned} \Delta E &= E_f - E_i \\ &= E_A + E_X - m_A \\ &= \sqrt{\mathbf{p}_A^2 + m_A^2} + \sqrt{\mathbf{p}_A^2 + m_X^2} - m_A \end{aligned}$$

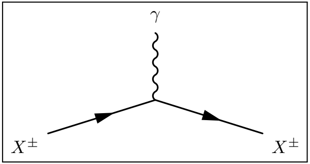
The limit case $\mathbf{p}_A \to 0$ gives $\Delta E = m_X$, so $\Delta E \geq m_X$
Therefore, Heisenberg says

$$\Delta E \Delta t \geq \frac{1}{2} \quad \to \Delta t \geq \frac{1}{2\Delta E} \quad \to \quad \Delta t_{unmeasurable} = \tau \leq \frac{1}{2m_X}$$

Massive propagators have limited *range*, R (remember $c = \hbar = 1$).

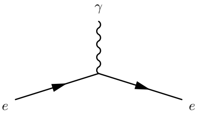
Quantum Electrodynamics (QED)

Electromagnetism mediated by the photon and described by QED.
Every QED interaction is based on this *vertex*:

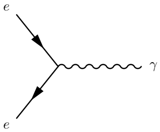


- the solid line ($X^{\pm}$) is any electromagnetically charged particle
- the squiggly line is a photon (γ)
- the *coupling constant* is $\alpha = \frac{1}{137}$

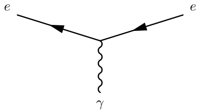
The vertex can be rotated to give other processes:



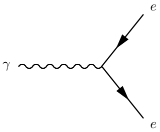
e^{-} scatter



$e^{+}e^{-}$ annihilation

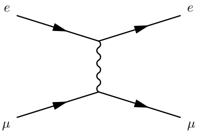


e^{+} scatter

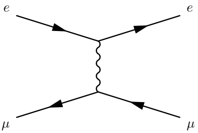


$e^{+}e^{-}$ pair production

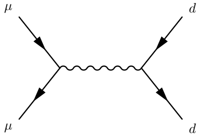
Some Examples of QED Interactions



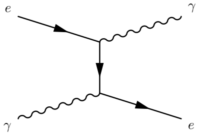
$e^- \mu^- \rightarrow e^- \mu^-$



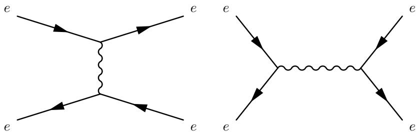
$e^- \mu^+ \rightarrow e^- \mu^+$



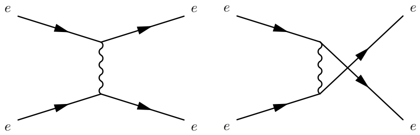
$\mu^+ \mu^- \rightarrow d\bar{d}$



$e^- \gamma \rightarrow e^- \gamma$



$e^+ e^- \rightarrow e^+ e^-$

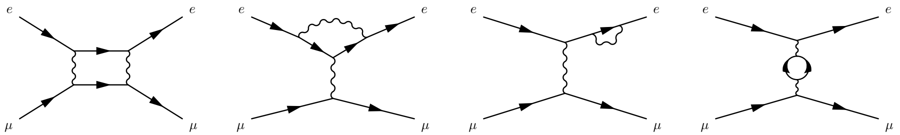


$e^- e^- \rightarrow e^- e^-$

Higher Order Diagrams

The previous examples are the *lowest order (LO)* diagrams for the processes. Every process has *higher order* diagrams.

Next-to-leading order (NLO) diagrams for $e^-\mu^- \to e^-\mu^-$ are:

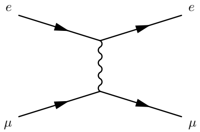


Higher order diagrams are constructed by adding additional *internal lines* without adding *external lines*.

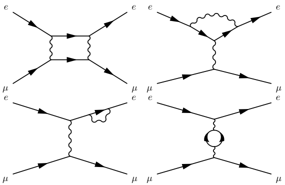
Note that each diagram is constructed of the fundamental QED vertex, each vertex with a 'strength' proportional to α.

Perturbation Theory

To calculate what happens in an interaction like $e^- \mu^- \rightarrow e^- \mu^-$, one must add the diagrams at every order:



$+$



$+ \cdots$

$\mathcal{O}(\alpha^2) \quad + \quad \mathcal{O}(\alpha^4) \quad + \cdots$

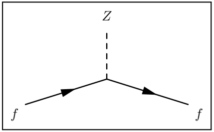
Because $\alpha < 1$, each higher order contributes a smaller amount to the result. Phew!

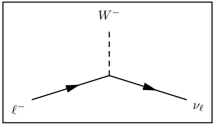
Weak Interactions

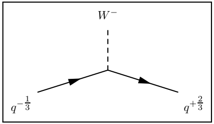
Weak interactions are mediated by W and Z

- the weak charge is rather complex...
- all fermions carry weak charge
- W boson couples charged leptons to neutrinos
- W boson can also change quark flavour

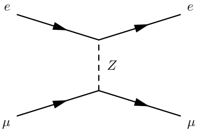
There are 3 weak interaction vertices:



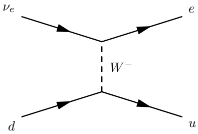




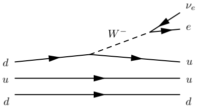
Weak Interaction Examples



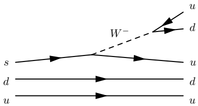
$$e^- + \mu^- \rightarrow e^- + \mu^-$$



$$d + \nu_e \rightarrow u + e^-$$



$$n \rightarrow p + e^- + \bar{\nu_e}$$



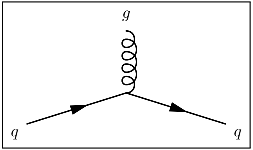
$$\Lambda \rightarrow p + \pi^-$$

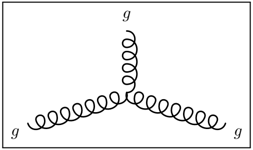
Quantum Chromodynamics (QCD)

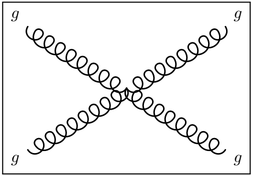
QCD describes the strong interaction mediated by the gluon

- the charge of the strong interaction is *colour*
- colour comes in 3 types: *red*, *green*, *blue* (plus anti-colours)
- only quarks and gluons carry colour charge

There are 3 fundamental QCD vertices:







The strong *coupling constant* is $\alpha_s \gtrsim 1$

Freedom and Confinement

The gluon carries colour, unlike the photon which does not carry electric charge, this has consequences...
Asymptotic Freedom:

- coupling constants: $\alpha_s \gtrsim 1$, while $\alpha < 1$
- thankfully, at small distances, α_s becomes < 1, so perturbation theory can be used for some QCD calculations
- this is called *asymptotic freedom* (quarks are "free to move around" inside a proton)

Confinement:

- no naturally occurring particles carry colour
- quarks are *confined* to bound states with no net colour charge
- particles composed of quarks are called *hadrons*

Hadron Classification

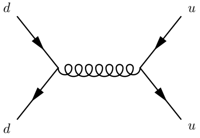
Hadron: a particle made of quarks is called a *hadron*.

- *Meson:* a hadron made of a quark-antiquark pair
- *Baryon:* a hadron made of three quarks or three antiquarks

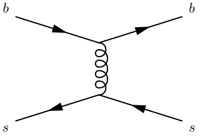
Examples:

	Quark Content	Spin	Charge	Mass (MeV)
Baryon				
p	uud	$1/2$	$+1$	938
$\bar{p}$	$\bar{u}\bar{u}\bar{d}$	$1/2$	-1	938
n	udd	$1/2$	0	939
Σ^0	uds	$1/2$	0	1192
Δ^+	uud	$3/2$	$+1$	1232
Δ^{++}	uuu	$3/2$	$+2$	1232
Meson				
π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	0	0	135
$\pi^\pm$	$u\bar{d}$, $d\bar{u}$	0	±1	140
$\rho^\pm$	$u\bar{d}$, $d\bar{u}$	1	±1	775
$K^\pm$	$u\bar{s}$, $s\bar{u}$	0	±1	494
$D^\pm$	$c\bar{d}$, $d\bar{c}$	0	±1	1869
$B^\pm$	$u\bar{b}$, $b\bar{u}$	0	±1	5279
ψ	$c\bar{c}$	1	0	3097
Υ	$b\bar{b}$	1	0	9460

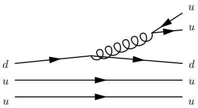
Example Diagrams



$d\bar{d} \rightarrow u\bar{u}$



$b\bar{s} \rightarrow b\bar{s}$



$\Delta^+ \rightarrow p + \pi^0$

Questions ?