Introduction to Elementary Particle Physics

1: What do we know so far

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Natural Units

$E^2 = p^2c^2 + m^2c^4$ and $\alpha = \frac{e^2}{\hbar c}$ $\rightarrow$ that's a lot of c's and $\hbar$'s...

$E^2 = p^2 + m^2$ and $\alpha = e^2$ $\rightarrow$ that's much prettier!

Particle physicists are a clever (or lazy) bunch,
we just say $c = \hbar = 1$ are *"Natural Units"*.

Implications:

- time is measured in "distance units"
- we use *electron volts (eV)* for mass, energy, and momentum

A handy number for conversion is: $\hbar c = 197\ MeV \cdot fm$

Natural Units II

Particle Physics

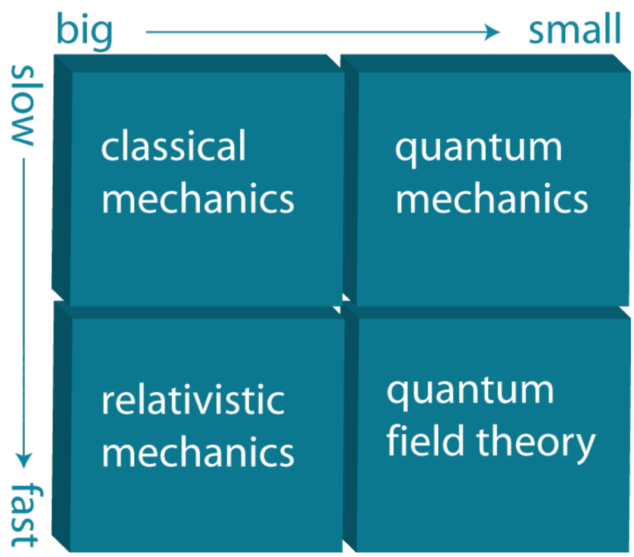
The base hypotheses of *particle physics* (in my opinion):

- the universe is made of a limited number of *fundamental*[1] *particles* responsible for all physical phenomena
- there are limited number of forces which interact with matter, these forces *may be* manifestations of *one universal force*
- all conservation laws reflect an underlying *symmetry of nature*

[1] fundamental (or elementary) means the particle has no internal structure

Quantum Field Theory

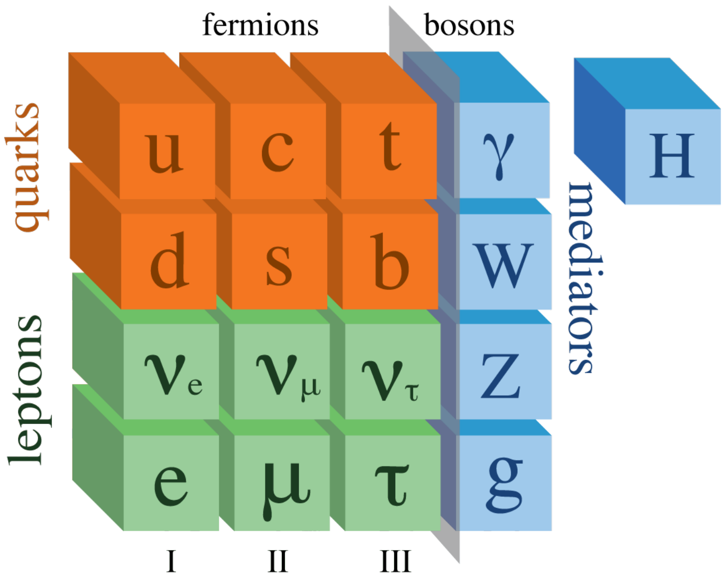
- QFT is quantum mechanics with relativistic mechanics.



Generally:

- Hons Rel. QM: general principles of QFT
- Hons. Particle Physics: use the results of QFT
- This Course: ba sic concepts of the Standard Model

The Standard Model



- The Standard Model is the theory of all 'known' fundamental particles and their interactions.
- It explains almost all known physical phenomena.
- It is a *Quantum Field Theory* (QFT).

From Lagrangian to Particles & Interactions

Classical Mechanics

- define the Lagrangian
  $L = \frac{1}{2}mv^2 - V(x)$
- use the Euler-Lagrange equation
  $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$
- get the equation of motion
  $F = ma$
- you get *Newton's Law*

Quantum Field Theory

- define the Lagrangian (density)
  $\mathcal{L}_f = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$
- use the Euler-Lagrange equation
  $\partial^\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu\psi)}\right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0$
- get the 'equation of motion'
  $(i\gamma^\mu\partial_\mu - m)\psi = 0$
- you get the *Dirac Equation*

The Standard Model is defined by a Lagrangian:

- the Lagrangian predicts the particles and their interactions

The Standard Model Lagrangian

$$
\begin{aligned}
&\mathcal{L} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \tfrac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \\
&\partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H - \tfrac{1}{2}m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \\
&M^2\phi^+\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
&W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - \\
&A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + \\
&g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - \\
&2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \tfrac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] - \\
&gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \tfrac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - \\
&W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - \\
&ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \tfrac{1}{4}g^2W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \tfrac{1}{4}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + \\
&(\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + \\
&W^-_\mu\phi^+) + \tfrac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - g^1s_w^2A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \\
&\bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_wA_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \tfrac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + \\
&(\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1 + \\
&\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_jC^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}\tfrac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \\
&\tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5e^\lambda)] + \tfrac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_jC_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar{u}^\lambda_jC_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1 + \\
&\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j] - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar{u}^\lambda_ju^\lambda_j) - \tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar{d}^\lambda_jd^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5u^\lambda_j) - \tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5d^\lambda_j) + \bar{X}^+(\partial^2 - \\
&M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \\
&\partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + \\
&igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) - \tfrac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H] + \tfrac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \\
&\tfrac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \tfrac{1}{2}igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]
\end{aligned}
$$

What is Force?

Force is transmitted by an exchange of particles.

The Four Forces

There are 4 known fundamental forces:

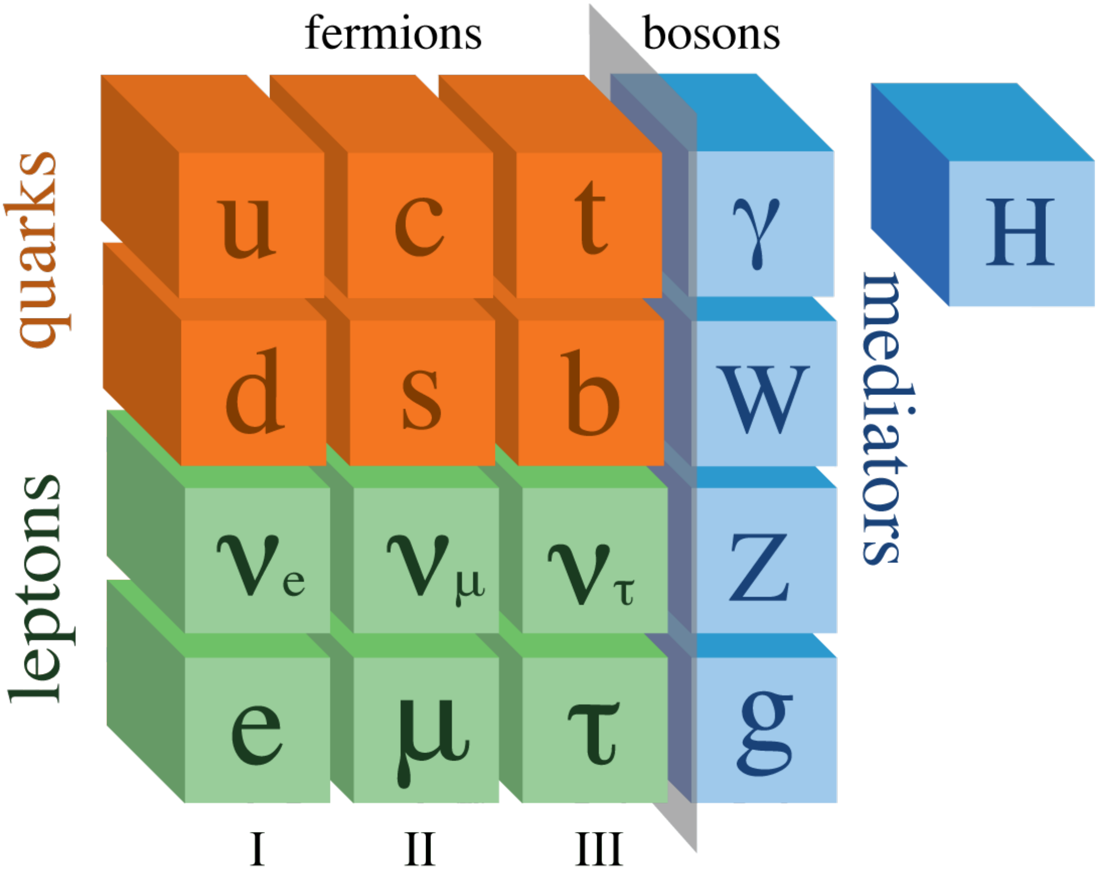
Force	Strength	Theory	Mediator
Strong	10	Quantum Chromodynamics (QCD)	gluon
Electromagnetic	10^{-2}	Quantum Electrodynamics (QED)	photon
Weak	10^{-13}	Glashow-Weinberg-Salam (GSW)	W and Z
Gravitation	10^{-42}	General Relativity	graviton

The Standard Model is a quantum field theory:

- QED, QCD, and GSW are all quantum field theories
- General relativity is not a quantum field theory

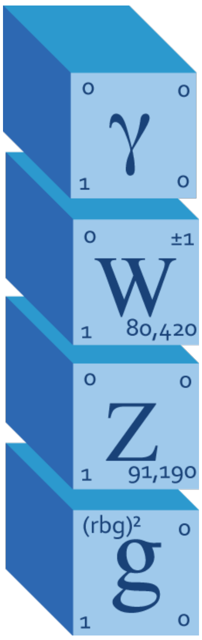
Therefore, gravity is not part of the Standard Model

The Standard Model



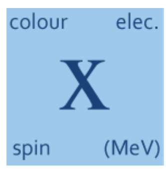
- Let's consider the different classes or particles ...

The Force Carriers (Mediators)

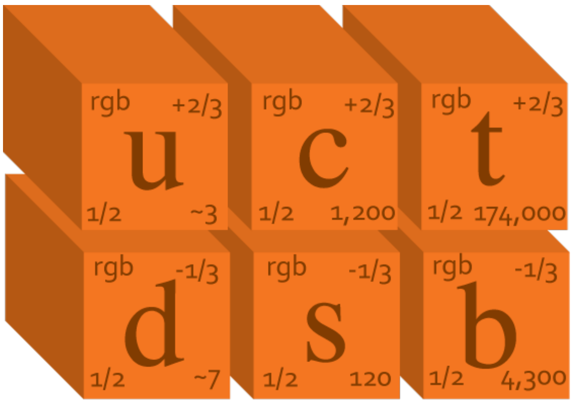


- photon (γ): transmits the *electromagnetic force* between 'electrically' charged $(+,-)$ particles, it has zero mass and carries 1 unit of spin
- W & Z: transmit the *weak force* between 'weakly' charged particles, have masses around 100 times the mass of the proton, and carry 1 unit of spin
- gluon (g): transmits the *strong force* between 'strongly' charged particles, it has zero mass and carries 1 unit of spin

All mediators are spin-1, so they are called *bosons*.

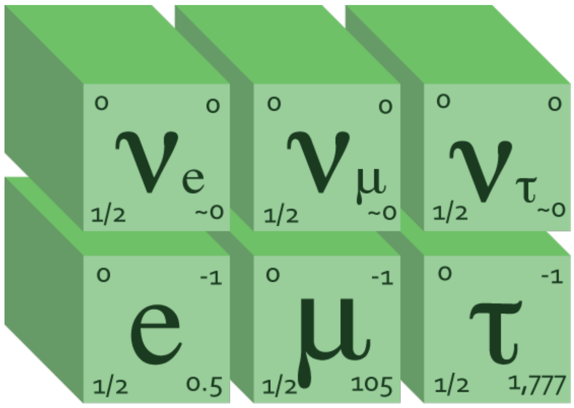


The Matter Particles



Quarks

- electrically, weakly, and strongly charged
- u and d form nuclear matter: proton $= (u, u, d)$



Leptons

- electrically and weakly charged
- e is our well known friend the electron

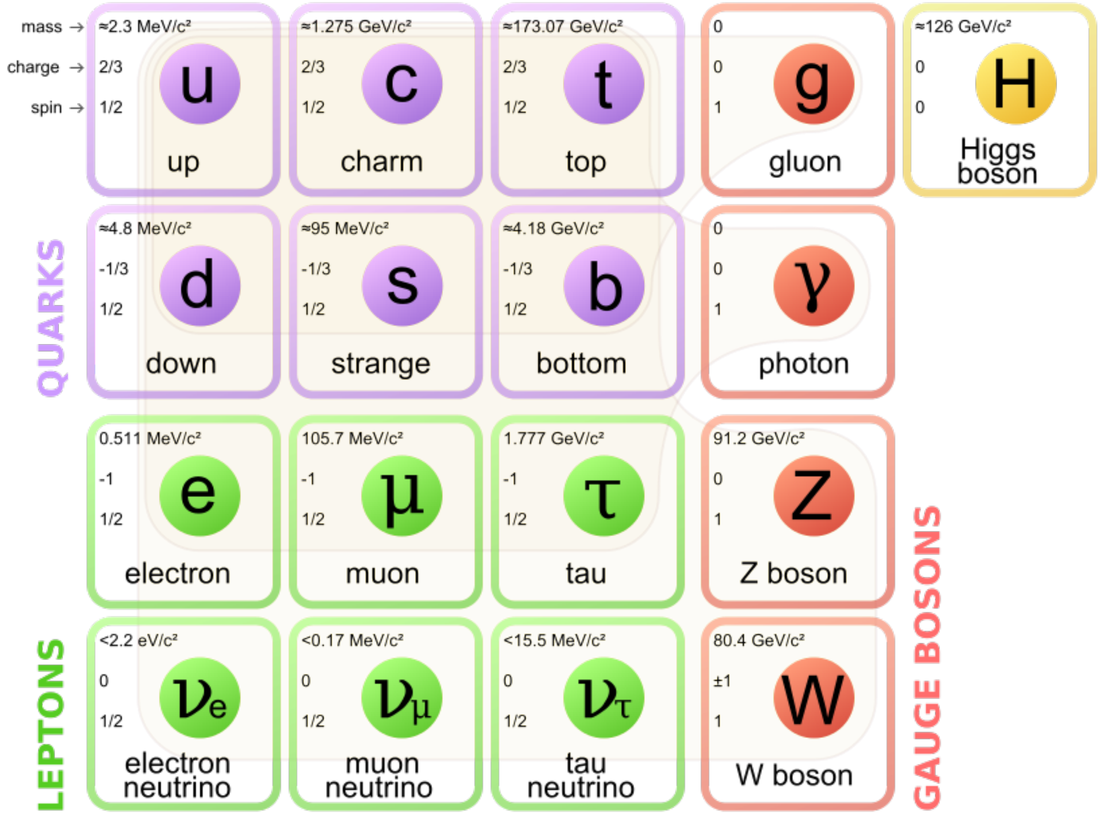
All matter particles are *fermions* (spin-$\frac{1}{2}$)

The Higgs

The Higgs gives mass to the fundamental particles.

- The theoretical structure of the SM requires that particles cannot have intrinsic mass
- we observe particles to have mass
- mass is energy, so can the mass actually come from an interaction
- we will (probably) not get to the details in this course

Standard Model Summary



*the photon and gluon do not directly couple to the Higgs...

The Small Matter of Antimatter

The Dirac equation, $(i\gamma^\mu\partial_\mu - m)\psi = 0$, is the equation of motion for a spin-$\frac{1}{2}$ particle, where ψ is a 4-component *"spinor"*:
$\psi = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$, this structure comes from combining Schrodingers equation with Special Relativity.

Finding the simplest stationary solutions leads to[2]:

$$\psi_A(t) = e^{-i\,m\cdot t}\begin{pmatrix} \psi_1(0) \\ \psi_2(0) \end{pmatrix} \quad \text{and} \quad \psi_B(t) = e^{+i\,m\cdot t}\begin{pmatrix} \psi_3(0) \\ \psi_4(0) \end{pmatrix}$$

How do you interpret $e^{+i\,m\cdot t}$?

[2]if interested, see Griffith's Chap. 8 for details

Antimatter 2

Antimatter 3

Standard Model Mysteries

- Well ... Gravity – Obviously
- related to gravity – Dark matter, Dark Energy
- The matter vs anti-matter Asymmetry of the universe
- There are 19 free parameters in the current theory
- There is no explanation for the observed structure (3 families)
- The behaviour of neutrinos
- Dark matter considerations lead us to believe that there are unknown (probably heavy) particles as yet undiscovered. For technical reasons this means that the Higgs mass is too low.

Energy-Momentum 4-vector

The *4-momentum* p^μ:

$$p^\mu \equiv (p^0, p^1, p^2, p^3) = (E, p_x, p_y, p_z) = (E, \mathbf{p})$$

$$E = \gamma m \quad \text{and} \quad \mathbf{p} = \gamma m \mathbf{v}$$

Using Einstein summation notation for p^μ:

$$p^\mu p_\mu \equiv E^2 - \mathbf{p}^2 = m^2$$

A particle's mass, m, is a *Lorentz Invariant*.

For massless particles, $E = |\mathbf{p}|$

Note: I will sometimes use the notation $p \cdot p$ to mean $p^\mu p_\mu$.

Conserved vs. Invariant

Invariant: the same in all inertial reference frames. A particle's mass is invariant, so it is sometimes called *invariant mass*:

$$p^\mu p_\mu = m^2$$

Conserved: the same before and after an interaction. In relativistic collisions[3], *4-momentum is conserved* in all particle collisions:

$$p_A^\mu + p_B^\mu = p_C^\mu + p_D^\mu$$

for a collision of $A + B \to C + D$.

[3] In classical collisions, mass, momentum, and energy are all independently conserved quantities - this is not true in relativistic collisions.

Collision Example: π^0 decay

A π^0 at rest decays to $\gamma + \gamma$, what is E_γ?

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$$
\begin{aligned}
p_\pi^\mu &= p_{\gamma_1}^\mu + p_{\gamma_2}^\mu \quad \text{(conservation of 4-momentum)} \\
p_\pi^2 &= p_{\gamma_1}^2 + p_{\gamma_2}^2 + 2\, p_{\gamma_1} \cdot p_{\gamma_2} \quad \text{(I dropped } ^\mu \text{ index)} \\
m_\pi^2 &= m_{\gamma_1}^2 + m_{\gamma_2}^2 + 2\,(E_{\gamma_1} E_{\gamma_2} - \mathbf{p}_{\gamma_1} \cdot \mathbf{p}_{\gamma_2}) \\
&\rightarrow m_\gamma = 0,\ E_\gamma = |\mathbf{p}_\gamma|,\ \mathbf{p}_{\gamma_1} = -\mathbf{p}_{\gamma_2},\ E_{\gamma_1} = E_{\gamma_2} \\
E_\gamma &= \frac{m_{\pi^-}}{2} \simeq 70\ MeV
\end{aligned}
$$

When solving kinematic problems:

- start with conservation of 4-momentum
- use 4-vector notation and exploit $p^\mu p_\mu = m^2$
- think about where 0's will be useful

Choose the right frame

Laboratory Frame: one particle is at rest. (also called *fixed target*)

Center-of-Momentum Frame: total 3-vector momentum is zero.

Example: What is the threshold energy for $p + p \to p + p + p + \bar{p}$ in a fixed target experiment?

Evaluate the invariants: $p^\mu p_\mu$ and $k^\mu k_\mu$ (p is total 4-momentum in lab frame, k is total 4-momentum in cm frame)

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Evaluate the invariants: $p^\mu p_\mu$ and $k^\mu k_\mu$ (p is total 4-momentum in lab frame, k is total 4-momentum in cm frame)

$$\begin{aligned} p &= (E+m, \mathbf{p}) \quad \text{(before collision)} \\ k &= (4m, 0) \quad \text{(after collision)} \\ p^\mu p_\mu &= k^\mu k_\mu \;\rightarrow\; (E+m)^2 - \mathbf{p}^2 = (4m)^2 \\ E &= 7m \quad \text{(after using } \mathbf{p}^2 = E^2 - m^2\text{)} \end{aligned}$$