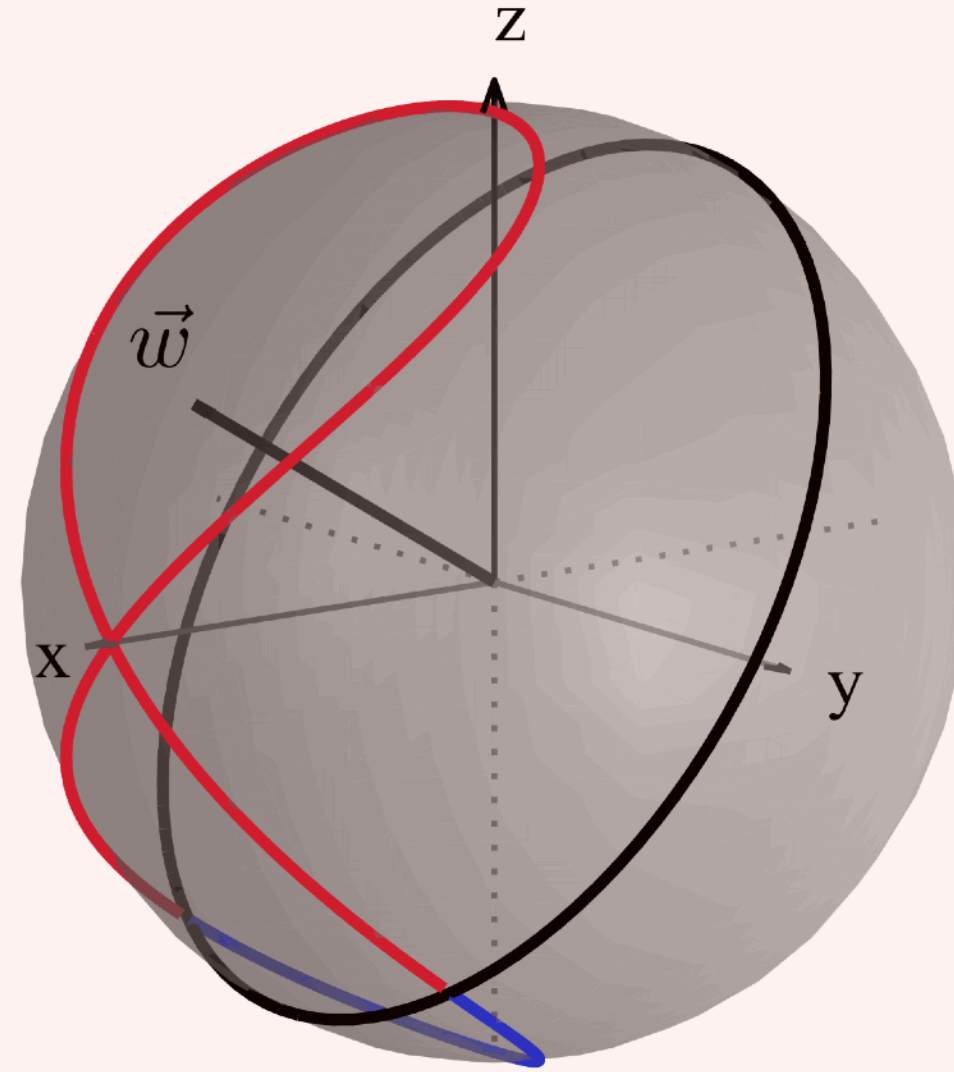




# Basic programming for quantum machine learning with Qiskit and PennyLane

NITheP Mini-school



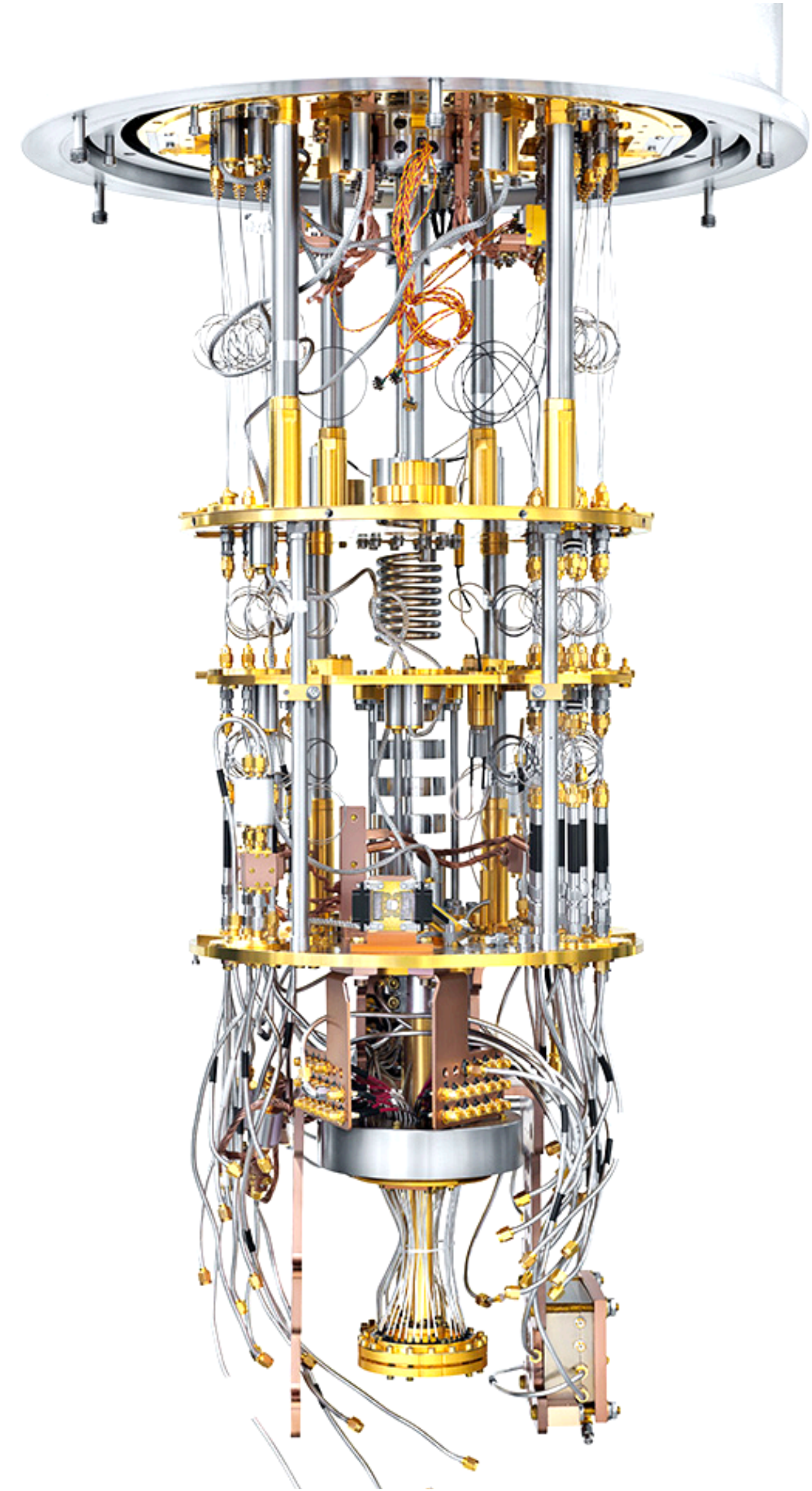
Amira Abbas  
15/09/2020



# Previously

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An introduction to variational circuits



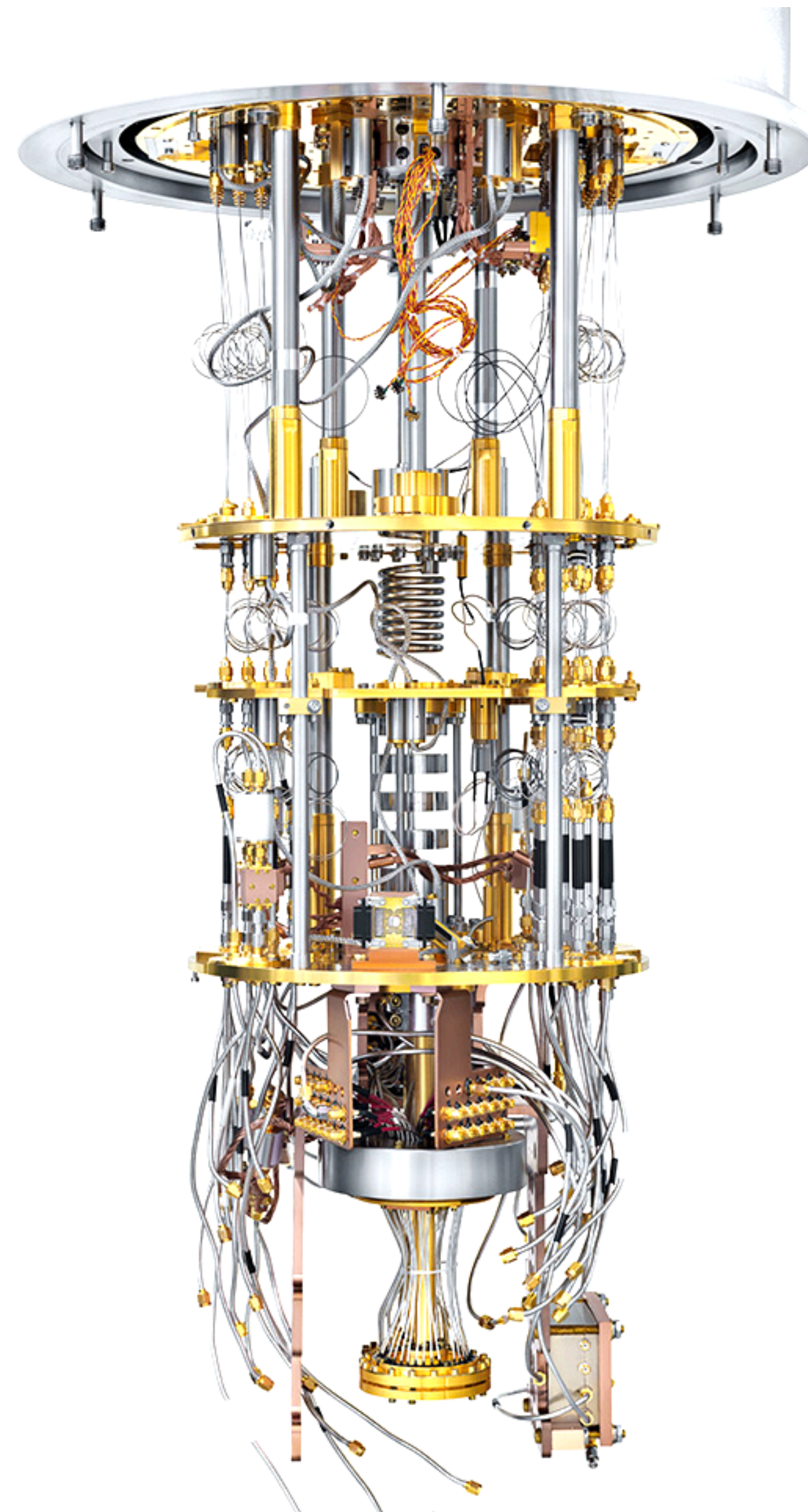


# Previously

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**An introduction to variational circuits**

**A simple model**





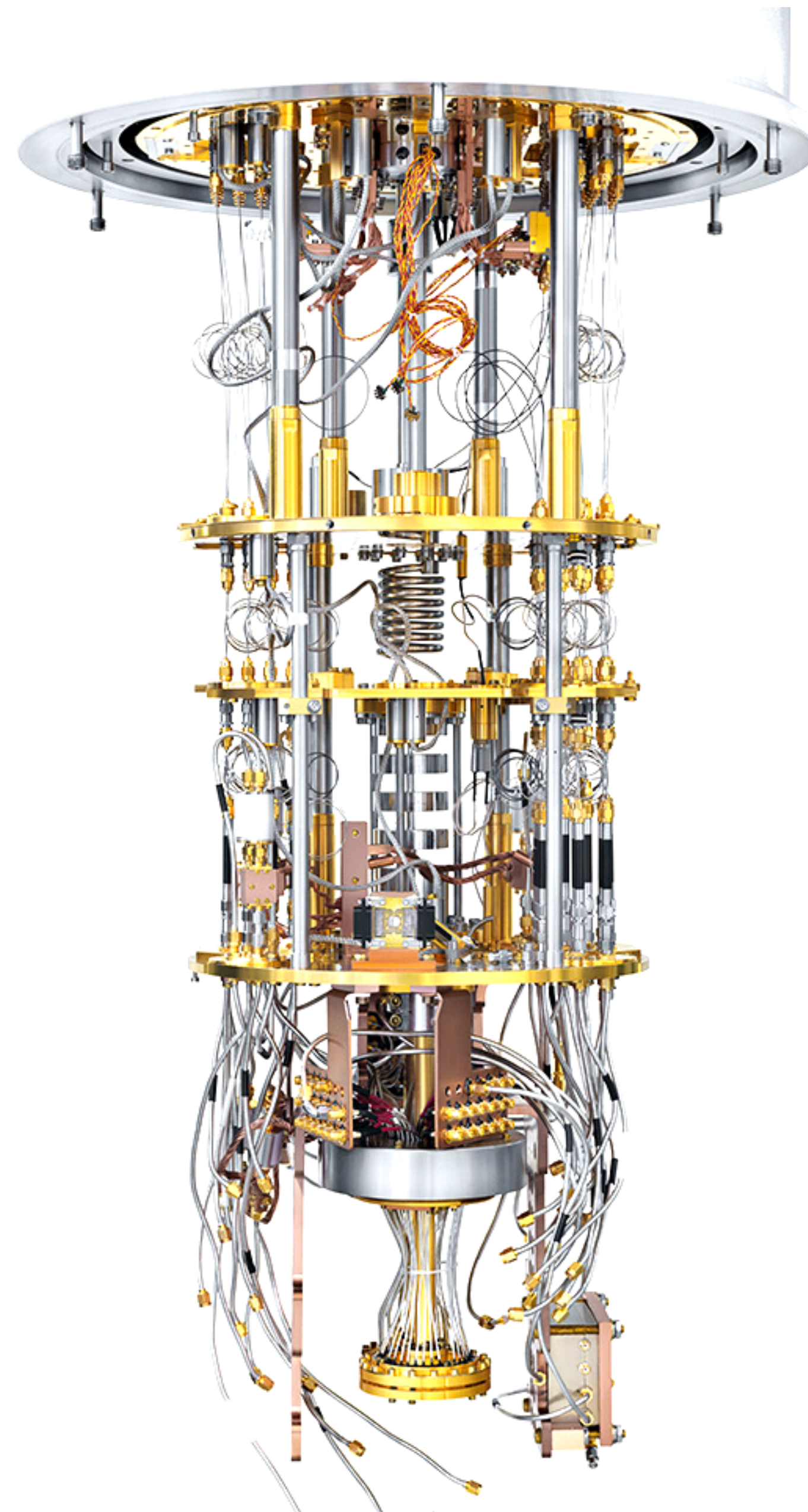
# Previously

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**An introduction to variational circuits**

**A simple model**

**Circuit optimisation**





# Previously

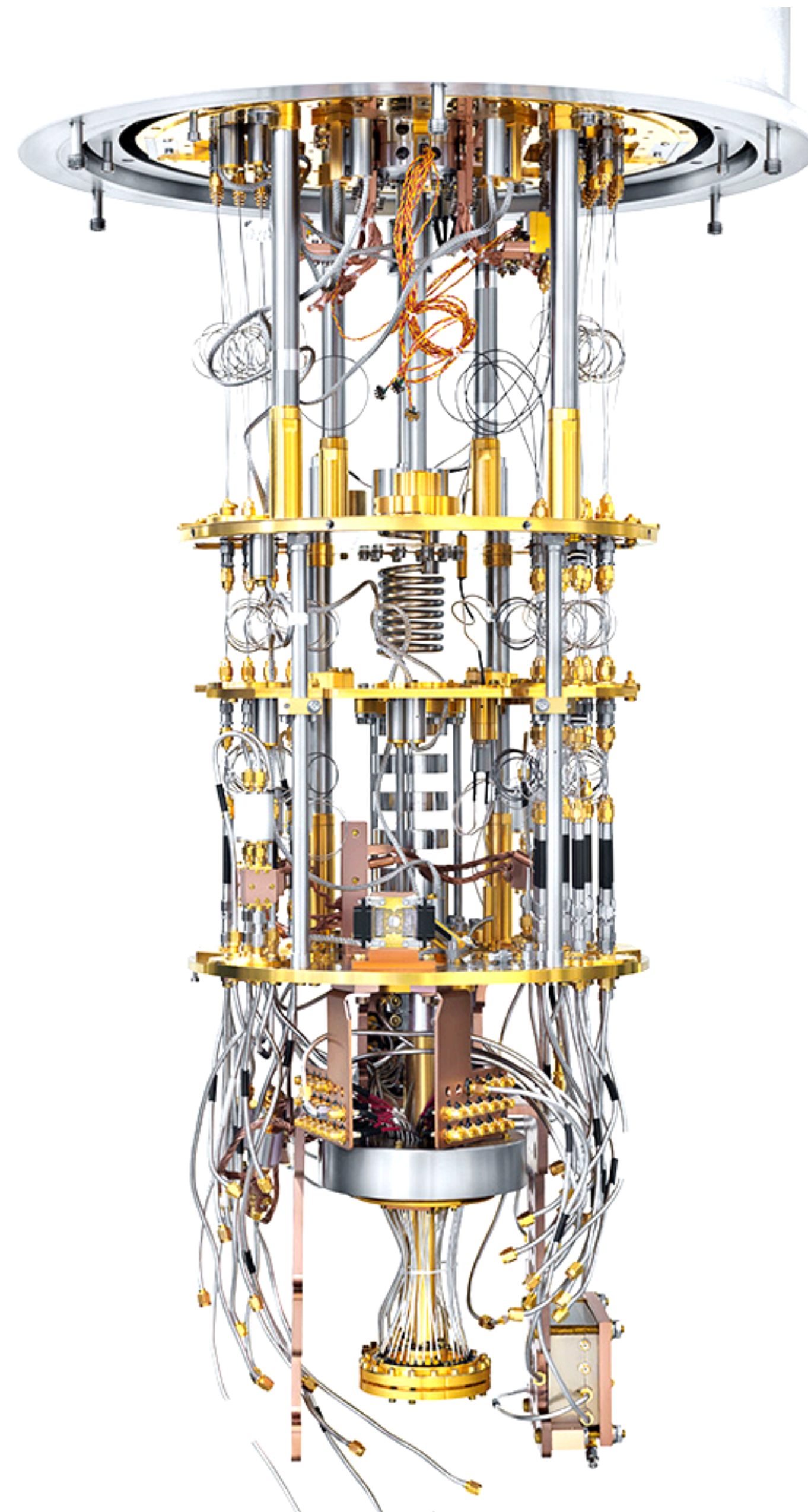
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**An introduction to variational circuits**

**A simple model**

**Circuit optimisation**

**Build the model with PennyLane**





# Agenda

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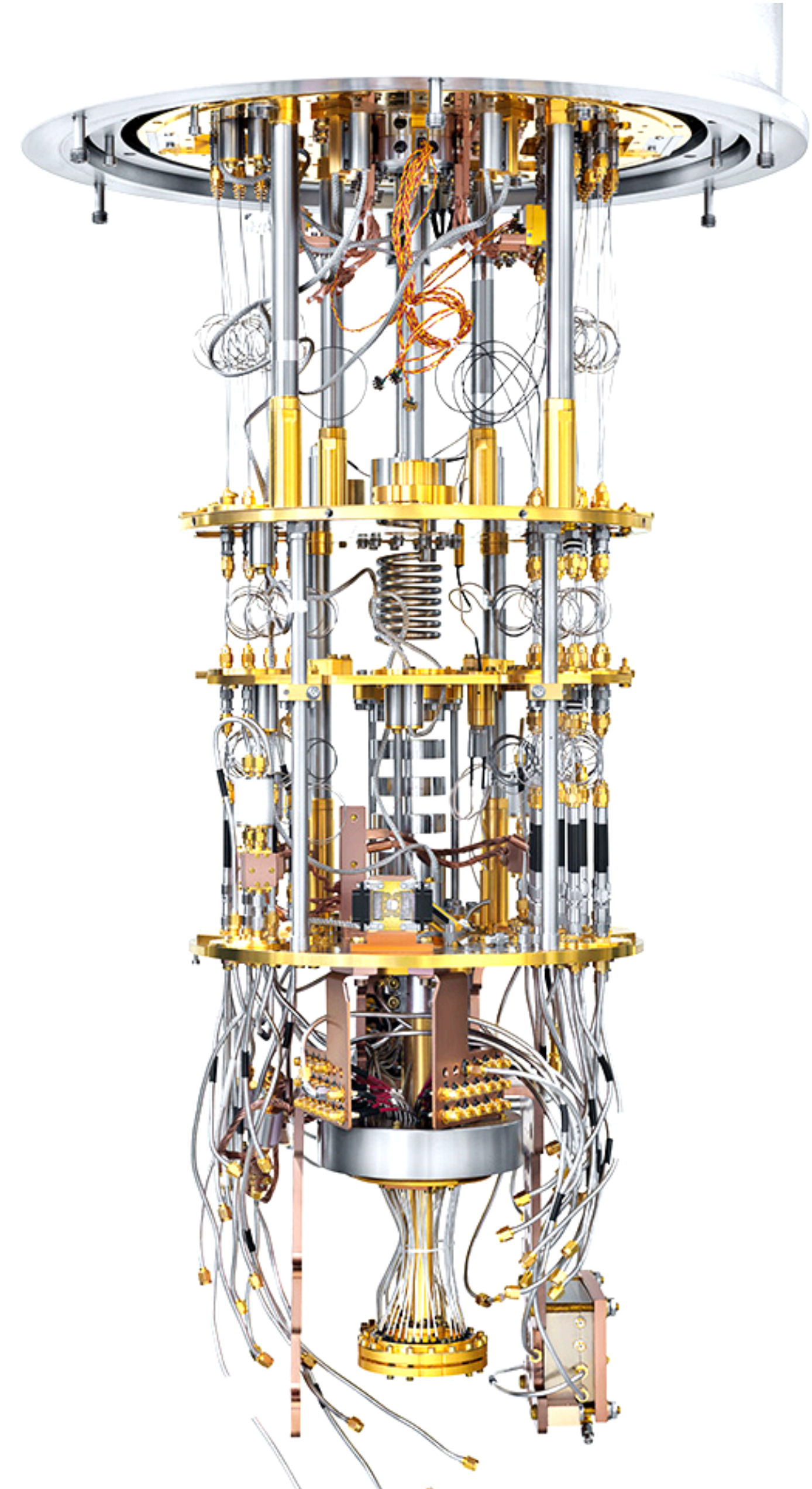
**An introduction to variational circuits**

**A simple model**

**Circuit optimisation**

**Build the model with PennyLane**

**Data encoding techniques**





# Agenda

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**An introduction to variational circuits**

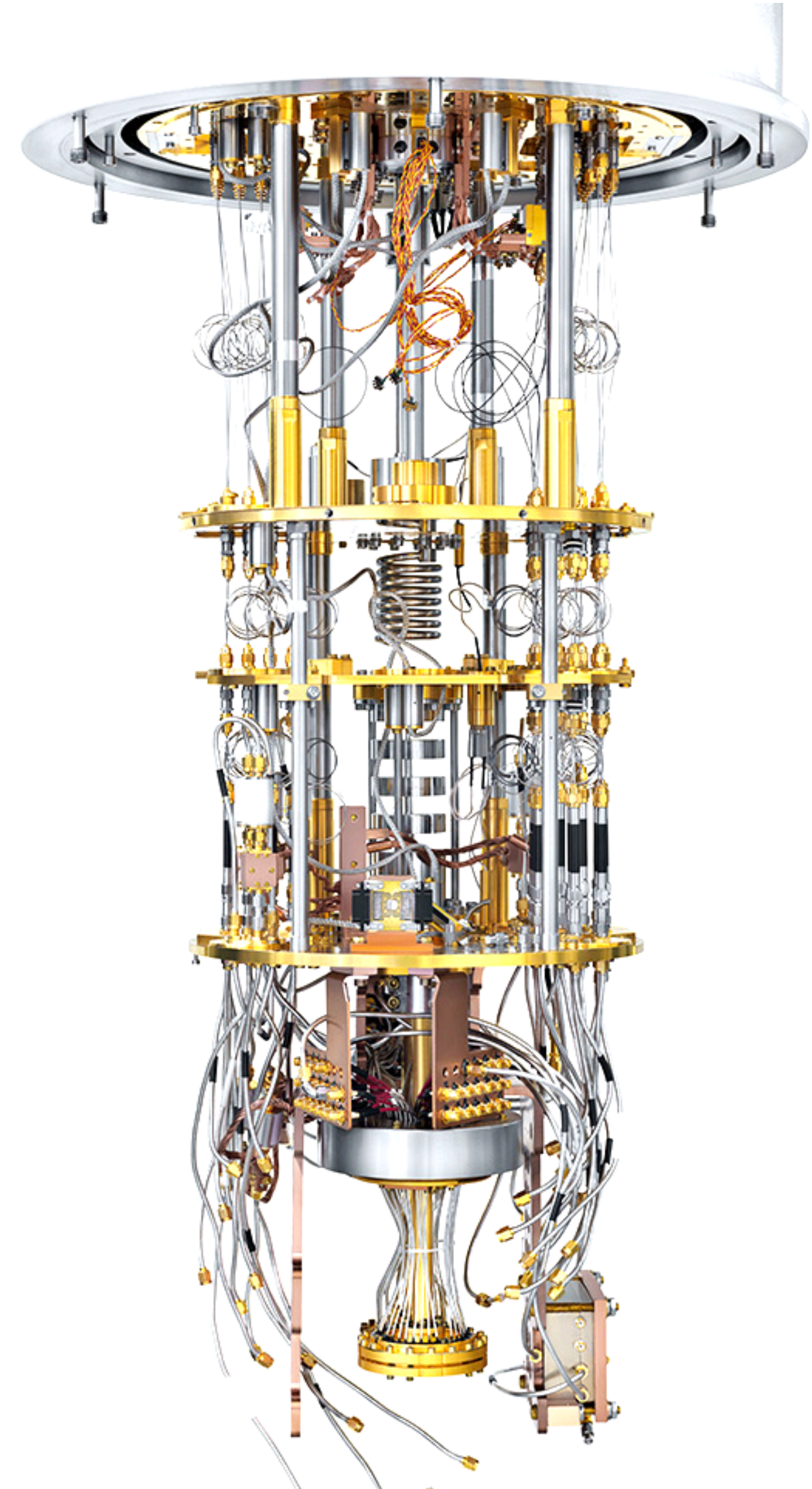
**A simple model**

**Circuit optimisation**

**Build the model with PennyLane**

**Data encoding techniques**

**Create circuits and encode data with PennyLane and Qiskit**



**Getting data into a quantum computer**



# Data encoding

---

- State preparation routines
  - ◆ Quantum circuits for general multi-qubit gates: <https://arxiv.org/pdf/quant-ph/0404089.pdf>
  - ◆ Quantum-state preparation with universal gate decompositions: <https://arxiv.org/pdf/1003.5760.pdf>
  - ◆ Quantum Networks for generating arbitrary quantum states: <https://arxiv.org/pdf/quant-ph/0407102.pdf>

# Data encoding

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- State preparation routines
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- Encoding classical data
  - ◆ Basis encoding - <https://arxiv.org/pdf/quant-ph/9807053.pdf>
  - ◆ Amplitude encoding - <https://arxiv.org/pdf/1703.10793.pdf>
  - ◆ Angle encoding - <https://arxiv.org/pdf/1711.11240.pdf>
  - ◆ Higher order embedding - <https://arxiv.org/pdf/1804.11326.pdf>
  - ◆ Variational/trained embedding - <https://arxiv.org/pdf/2001.03622.pdf>



# Data encoding

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- State preparation routines
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  - ◆ Variational/trained embedding - <https://arxiv.org/pdf/2001.03622.pdf>
- Open research question

# Basis encoding

---



# Basis encoding

---

Binary dataset  $D$  where each  $x^m \in D$  is a binary string of the form  $x^m = (b_1^m, \dots, b_N^m)$  with  $b_i^m \in \{0,1\}$  for  $i = 1, \dots, N$ .

# Basis encoding

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Binary dataset  $D$  where each  $x^m \in D$  is a binary string of the form  $x^m = (b_1^m, \dots, b_N^m)$  with  $b_i^m \in \{0,1\}$  for  $i = 1, \dots, N$ .

$$|D\rangle = \frac{1}{\sqrt{M}} \sum_{m=1}^M |x^m\rangle$$



# Basis encoding

---

For example,

$$x^1 = (0, 1)^T$$
$$x^2 = (1, 1)^T$$

# Basis encoding

---

For example,

$$\begin{aligned}x^1 &= (0, 1)^T \rightarrow |01\rangle \\x^2 &= (1, 1)^T \rightarrow |11\rangle\end{aligned}$$

# Basis encoding

---

For example,

$$\begin{aligned}x^1 &= (0, 1)^T \\ x^2 &= (1, 1)^T\end{aligned}$$

Creating a superposition of states using 2 qubits:

$$\frac{1}{\sqrt{4}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$



# Basis encoding

---

For example,

$$\begin{aligned}x^1 &= (0, 1)^T \\ x^2 &= (1, 1)^T\end{aligned}$$

Creating a superposition of states using 2 qubits:

$$\frac{1}{\sqrt{4}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$
$$\left(\frac{1}{\sqrt{4}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{4}}\right)^T$$

# Basis encoding

---

For example,

$$\begin{aligned}x^1 &= (0, 1)^T \\ x^2 &= (1, 1)^T\end{aligned}$$

Creating a superposition of states using 2 qubits:

$$\cancel{|00\rangle} + \frac{1}{\sqrt{2}} |01\rangle + \cancel{|10\rangle} + \frac{1}{\sqrt{2}} |11\rangle$$
$$(0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})^T$$

# Basis encoding

---

For example,

$$\begin{aligned}x^1 &= (0, 1)^T \\ x^2 &= (1, 1)^T\end{aligned}$$

Creating a superposition of states using 2 qubits:

$$|D\rangle = \frac{1}{\sqrt{2}} |01\rangle + \frac{1}{\sqrt{2}} |11\rangle$$



# Basis encoding

---

For example,

$$x^1 = (01, 01)^T$$
$$x^2 = (11, 10)^T$$

Creating a superposition of states using 4 qubits:

# Basis encoding

---

For example,

$$\begin{aligned}x^1 &= (01, 01)^T \\ x^2 &= (11, 10)^T\end{aligned}$$

Creating a superposition of states using 4 qubits:

$$|D\rangle = \frac{1}{\sqrt{2}} |0101\rangle + \frac{1}{\sqrt{2}} |1110\rangle$$

# Basis encoding

---

For example,

$$\begin{aligned}x^1 &= (01, 01)^T \\ x^2 &= (11, 10)^T\end{aligned}$$

Creating a superposition of states using 4 qubits:

$$|D\rangle = \frac{1}{\sqrt{2}} |0101\rangle + \frac{1}{\sqrt{2}} |1110\rangle$$

$$(0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0)^T$$



# Basis encoding

---

- Amplitude vector becomes very sparse in higher dimensions
- Most freedom to do computation
- Schemes are not always efficient
- Ventura, Dan, and Tony Martinez. "Quantum associative memory." *Information Sciences* 124.1-4 (2000): 273-296.



# Amplitude encoding

---

# Amplitude encoding

---

For example,

$$x^1 = (0.888, -1.25)^T$$

$$x^2 = (-0.23, 0.992)^T$$

# Amplitude encoding

---

For example,

$$x^1 = (0.888, -1.25)^T$$
$$x^2 = (-0.23, 0.992)^T$$



$$(0.888, -1.25, -0.23, 0.992)^T$$

$$|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle$$



# Amplitude encoding

---

For example,

$$x^1 = (0.888, -1.25)^T$$
$$x^2 = (-0.23, 0.992)^T$$



$$\frac{1}{\sqrt{4}}(0.888, -1.25, -0.23, 0.992)^T$$
$$|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle$$



# Amplitude encoding

---

$$|D\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} x_i |i\rangle$$

# Amplitude encoding

---

$$|D\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} x_i |i\rangle \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.888 \\ -1.25 \\ -0.23 \\ 0.992 \end{bmatrix}$$

# Amplitude encoding

---

$$|D\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} x_i |i\rangle \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.888 \\ -1.25 \\ -0.23 \\ 0.992 \end{bmatrix} \quad 2^n = 4$$



# Amplitude encoding

---

$$|D\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} x_i |i\rangle \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.888 \\ -1.25 \\ -0.23 \\ 0.992 \end{bmatrix} \quad 2^n = 4, n = 2$$

# Amplitude encoding

---

$$|D\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} x_i |i\rangle \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x^{1024} \end{bmatrix} \quad 2^n = 1024$$



# Amplitude encoding

---

$$|D\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} x_i |i\rangle \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x^{1024} \end{bmatrix} \quad 2^n = 1024, n = 10$$

# Angle encoding

---

# Angle encoding

---

For example,

$$x^1 = \begin{bmatrix} 0.888 \\ -1.25 \end{bmatrix} \quad x^2 = \begin{bmatrix} -0.23 \\ 0.992 \end{bmatrix}$$

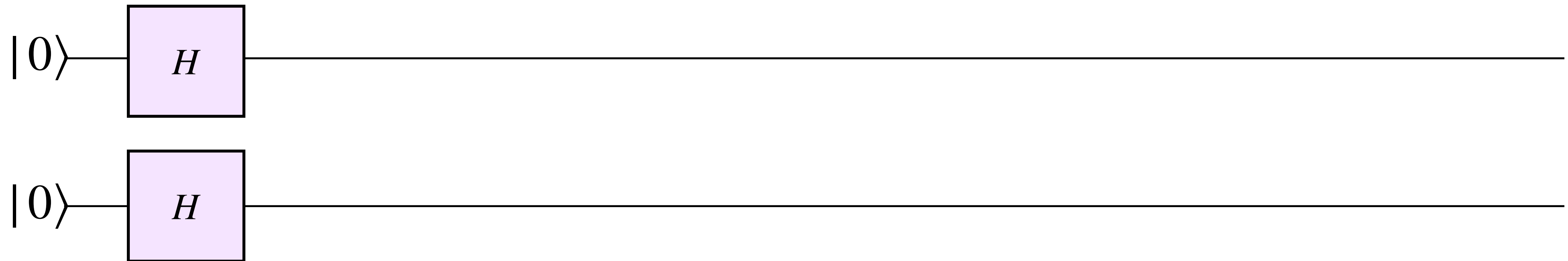


# Angle encoding

---

For example,

$$x^1 = \begin{bmatrix} 0.888 \\ -1.25 \end{bmatrix} \quad x^2 = \begin{bmatrix} -0.23 \\ 0.992 \end{bmatrix}$$

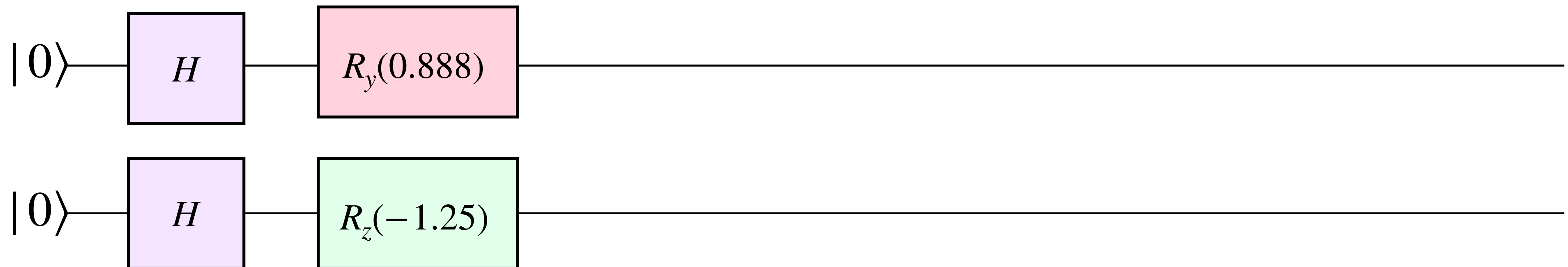


# Angle encoding

---

For example,

$$x^1 = \begin{bmatrix} 0.888 \\ -1.25 \end{bmatrix} \quad x^2 = \begin{bmatrix} -0.23 \\ 0.992 \end{bmatrix}$$

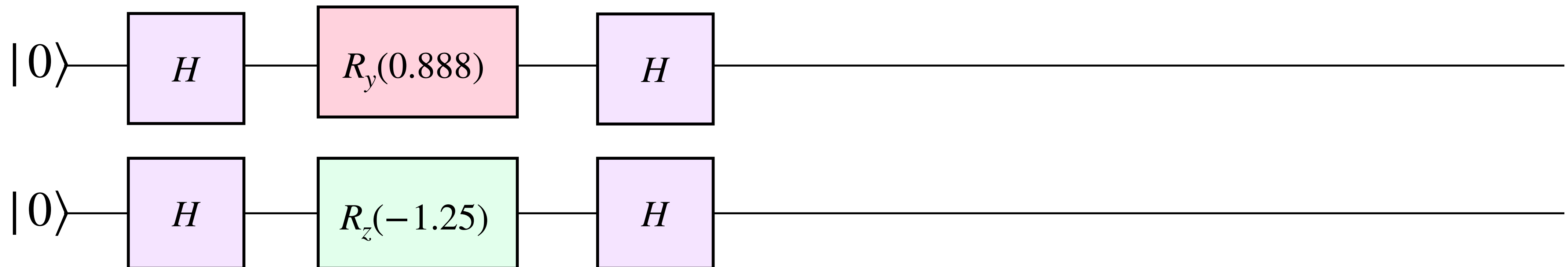


# Angle encoding

---

For example,

$$x^1 = \begin{bmatrix} 0.888 \\ -1.25 \end{bmatrix} \quad x^2 = \begin{bmatrix} -0.23 \\ 0.992 \end{bmatrix}$$



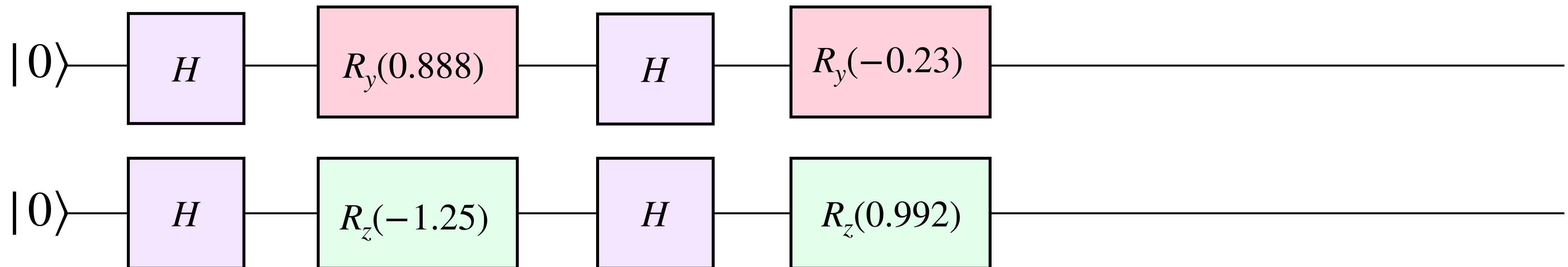


# Angle encoding

---

For example,

$$x^1 = \begin{bmatrix} 0.888 \\ -1.25 \end{bmatrix} \quad x^2 = \begin{bmatrix} -0.23 \\ 0.992 \end{bmatrix}$$

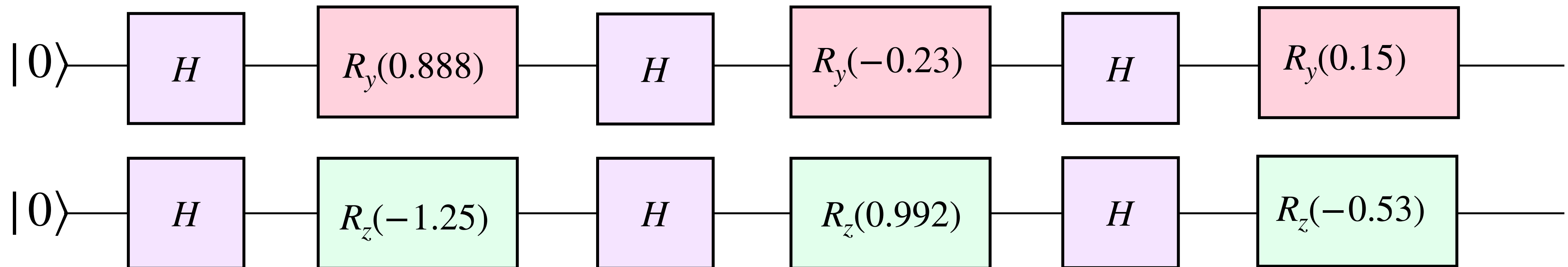


# Angle encoding

---

For example,

$$x^1 = \begin{bmatrix} 0.888 \\ -1.25 \end{bmatrix} \quad x^2 = \begin{bmatrix} -0.23 \\ 0.992 \end{bmatrix} \quad x^3 = \begin{bmatrix} 0.15 \\ -0.53 \end{bmatrix}$$

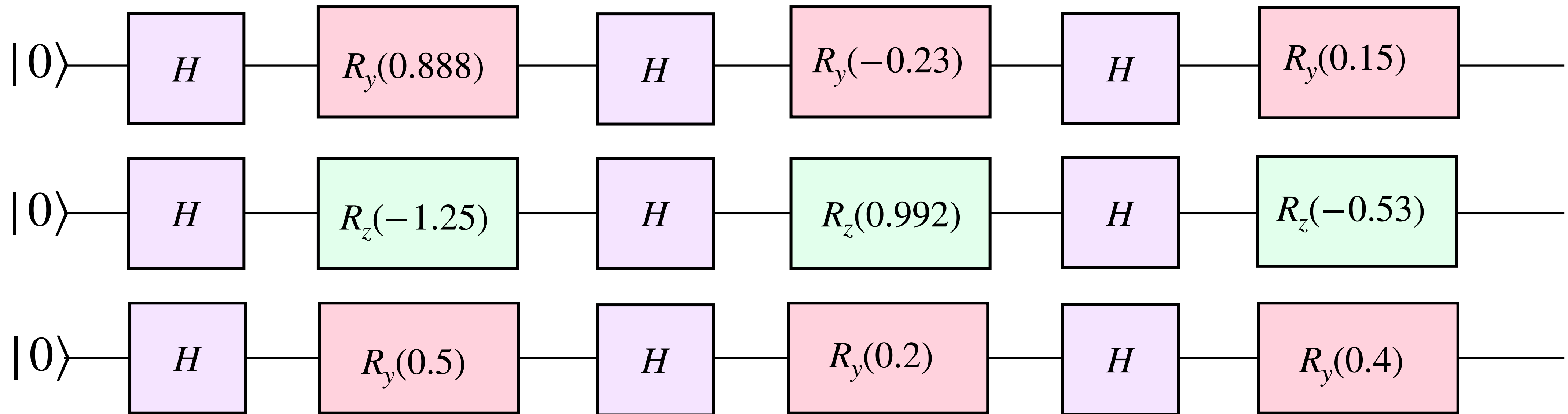


# Angle encoding

---

For example,

$$x^1 = \begin{bmatrix} 0.888 \\ -1.25 \\ 0.5 \end{bmatrix} \quad x^2 = \begin{bmatrix} -0.23 \\ 0.992 \\ 0.2 \end{bmatrix} \quad x^3 = \begin{bmatrix} 0.15 \\ -0.53 \\ 0.4 \end{bmatrix}$$





# Angle encoding

---

- Simple, intuitive
- Inexpensive
- Schuld, Maria, Ryan Sweke, and Johannes Jakob Meyer. "The effect of data encoding on the expressive power of variational quantum machine learning models." arXiv preprint arXiv:2008.08605 (2020).

# Quantum feature maps

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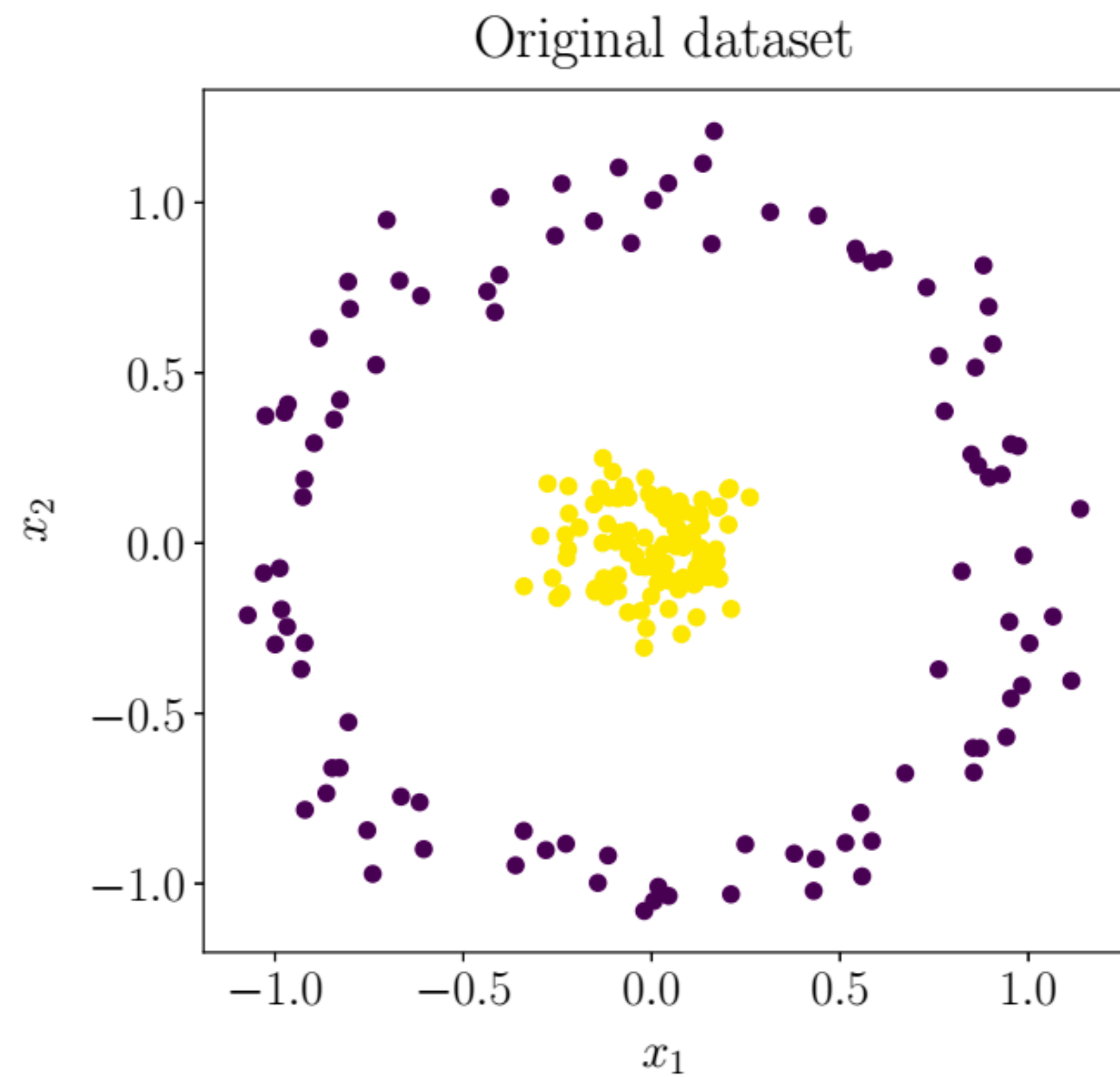
# Feature maps

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# Feature maps

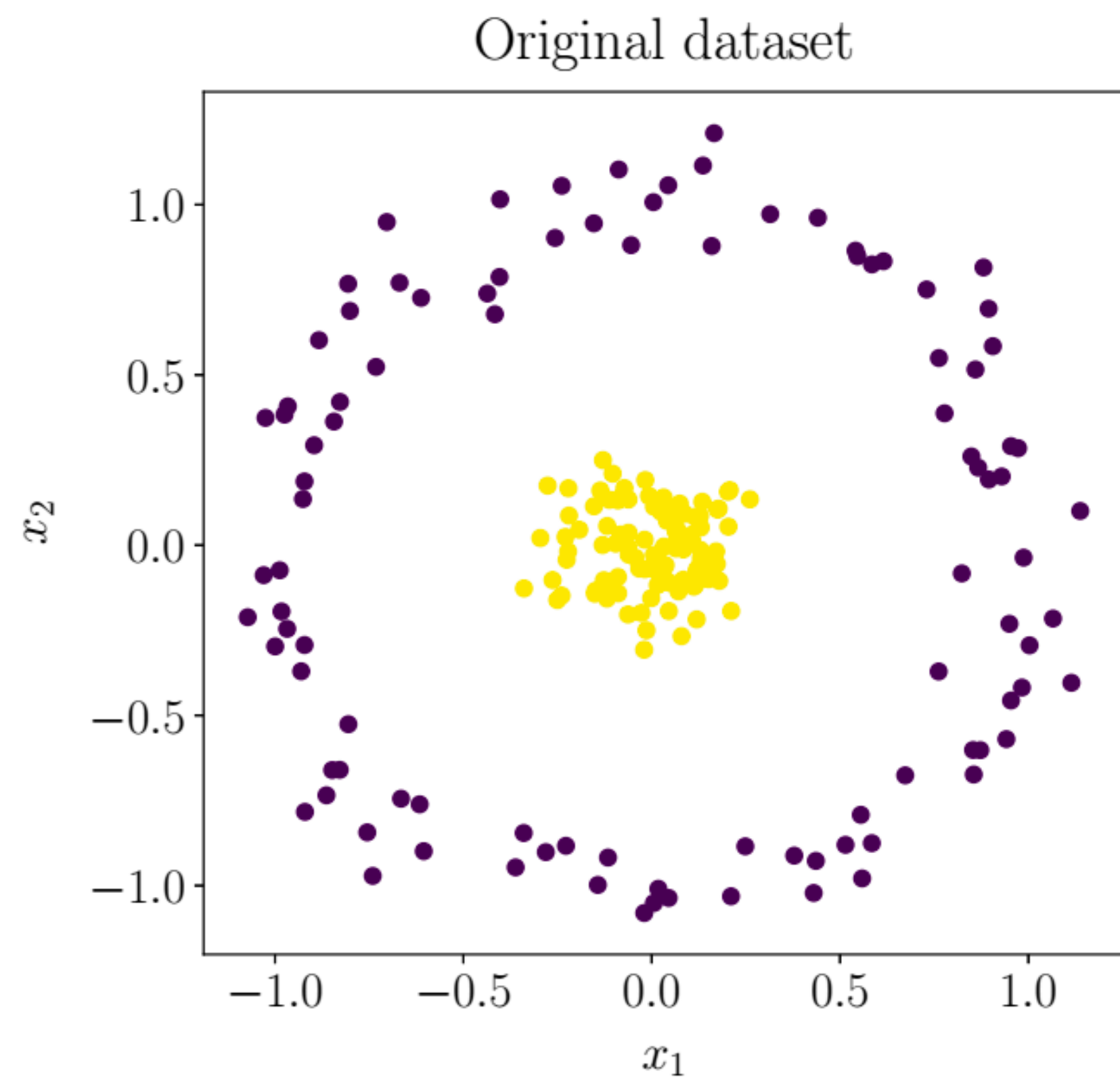
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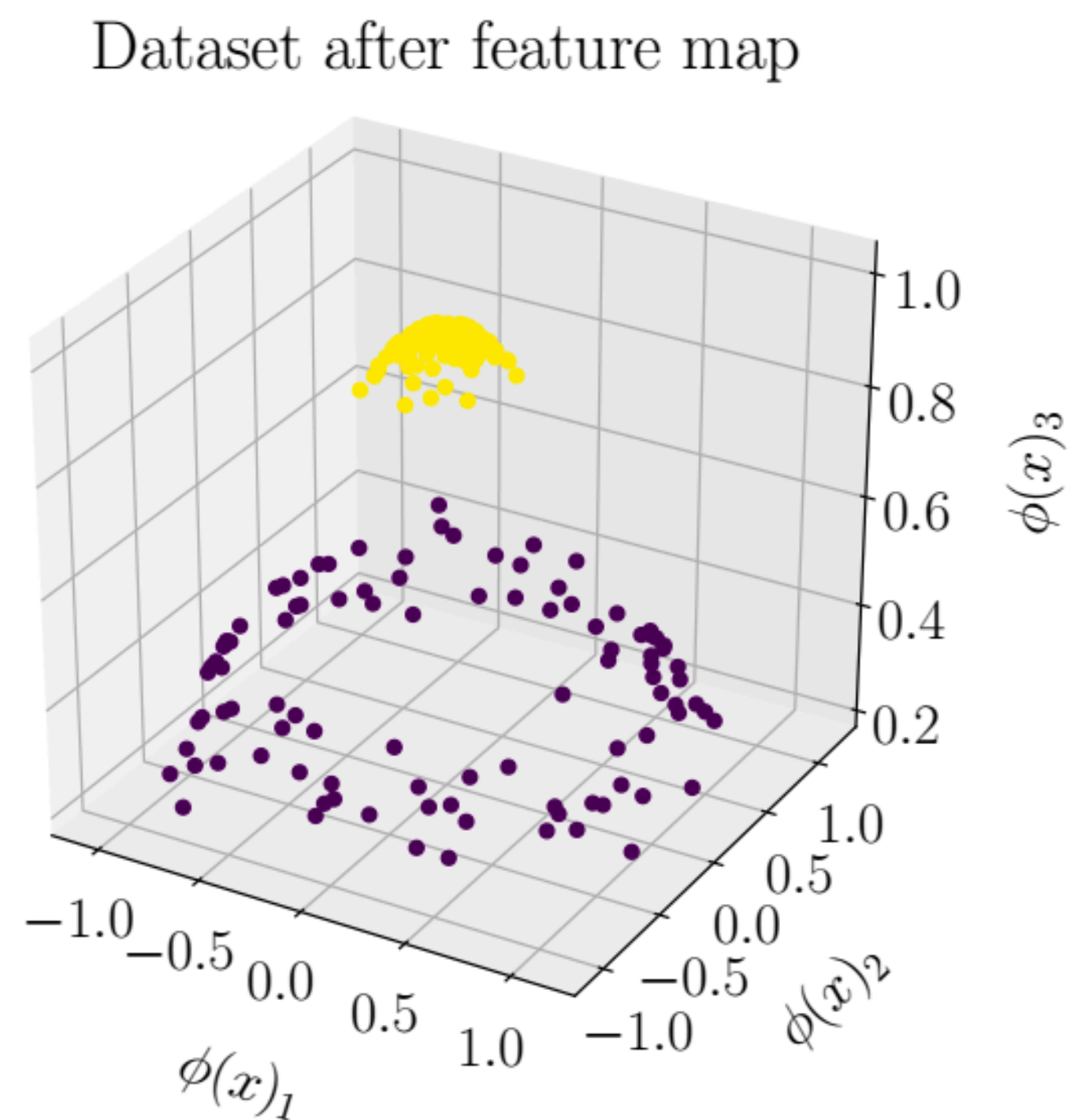
$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

# Feature maps

---

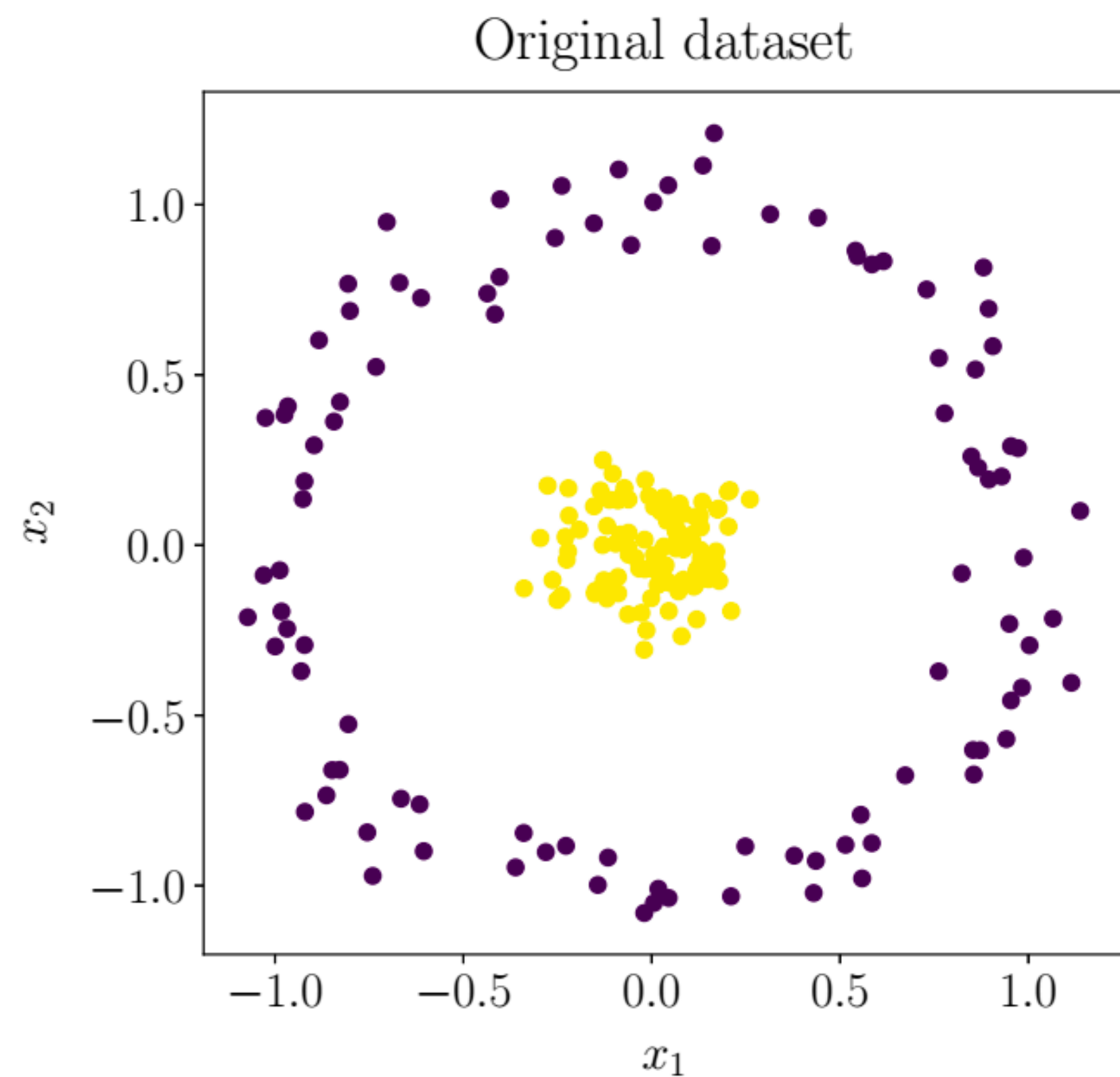


$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

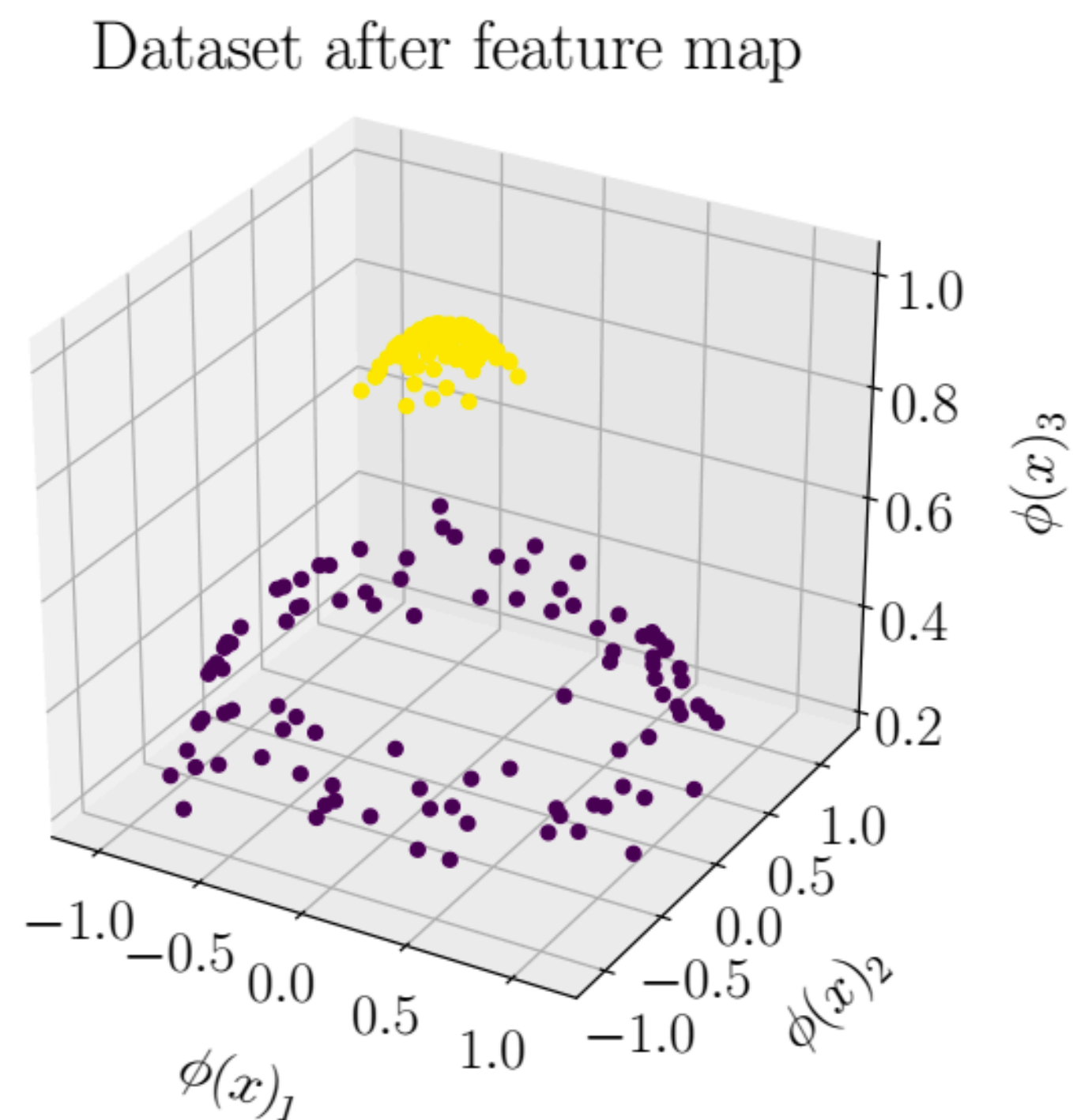


# Feature maps

---



$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

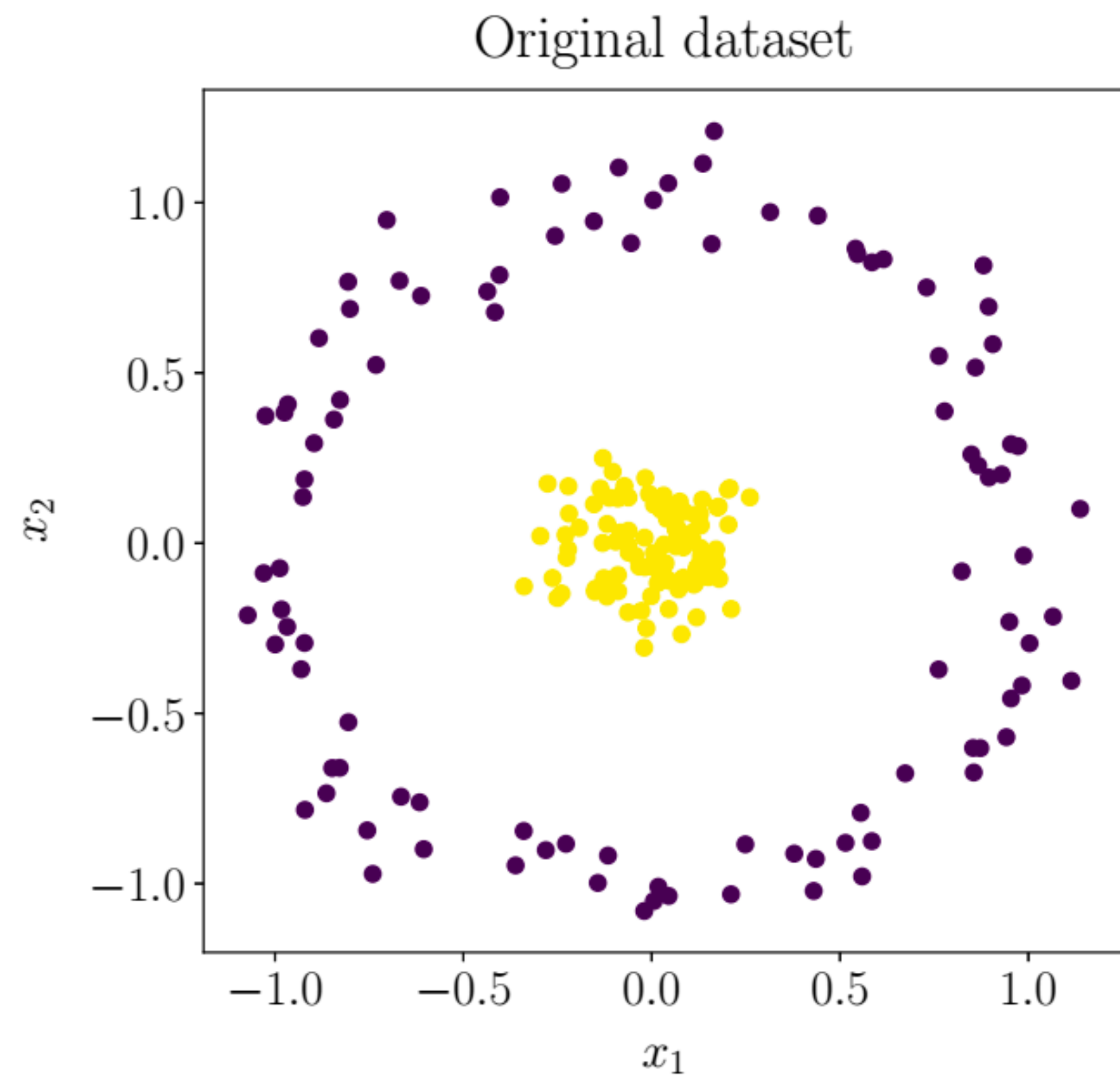


$$\phi(x) = \begin{bmatrix} \phi(x_1) \\ \phi(x_2) \\ \phi(x_3) \end{bmatrix}$$

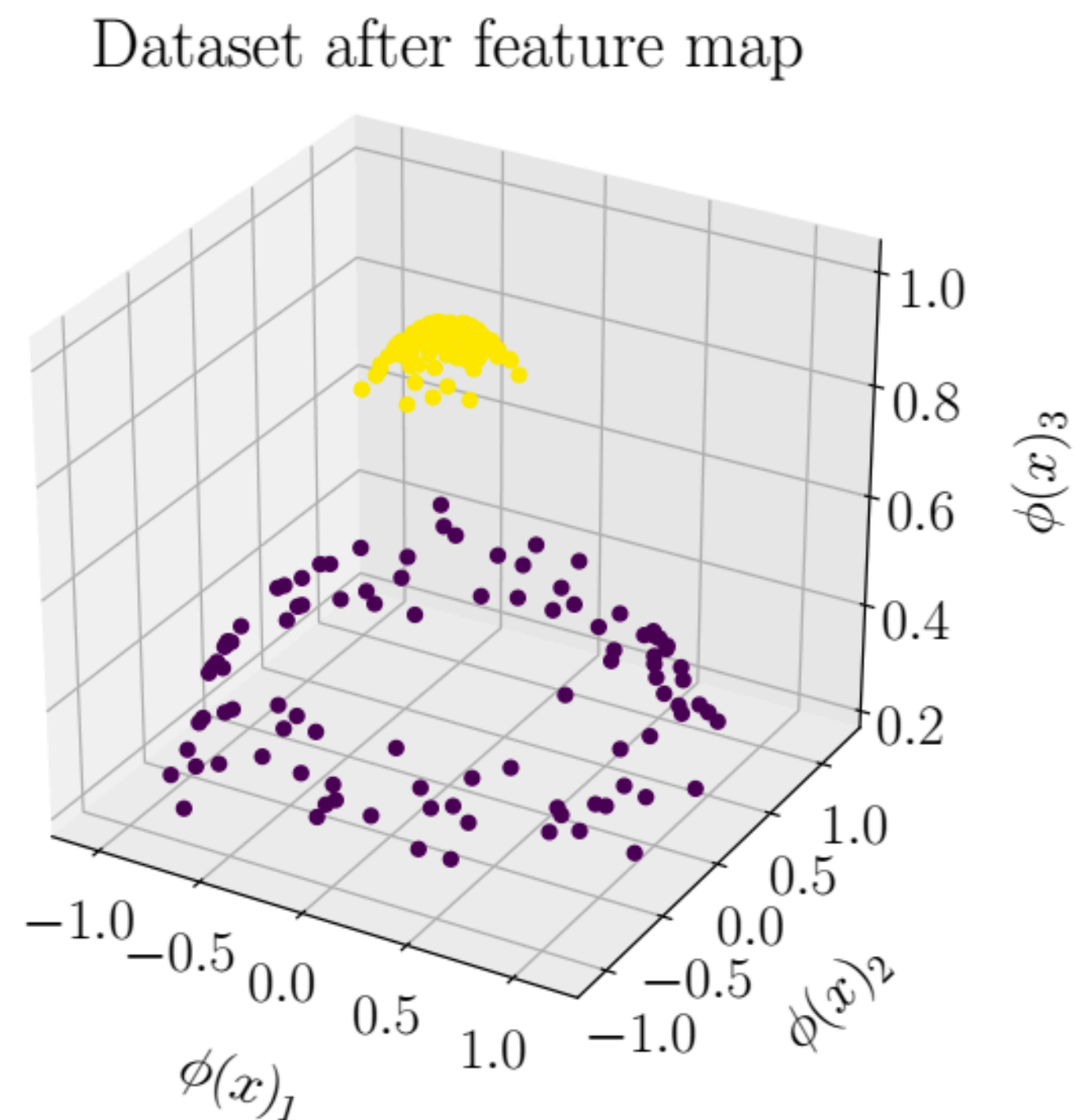


# Feature maps

---



$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$\phi(x) = \begin{bmatrix} \phi(x_1) \\ \phi(x_2) \\ \phi(x_3) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_1 * x_2 \end{bmatrix}$$

# Quantum feature maps

---

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

# Quantum feature maps

---

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \longrightarrow |x\rangle = \begin{bmatrix} \phi(x_1) \\ \phi(x_2) \\ \vdots \end{bmatrix}$$



# **Higher order embedding**



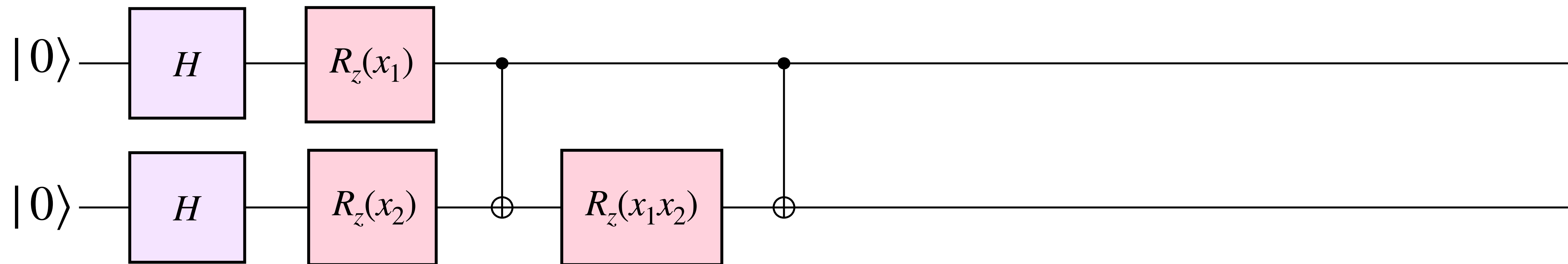
# Higher order embedding

---

For  $x^1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ :

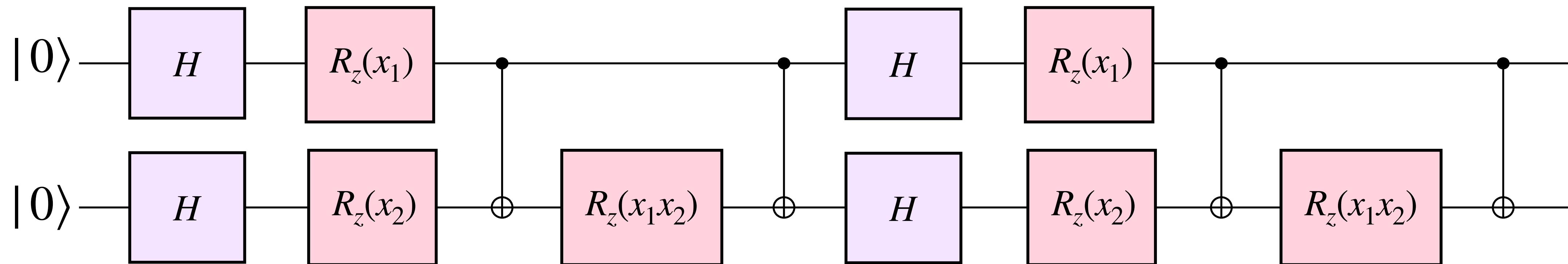
# Higher order embedding

For  $x^1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ :



# Higher order embedding

For  $x^1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ :



Conjectured to be difficult to simulate classically



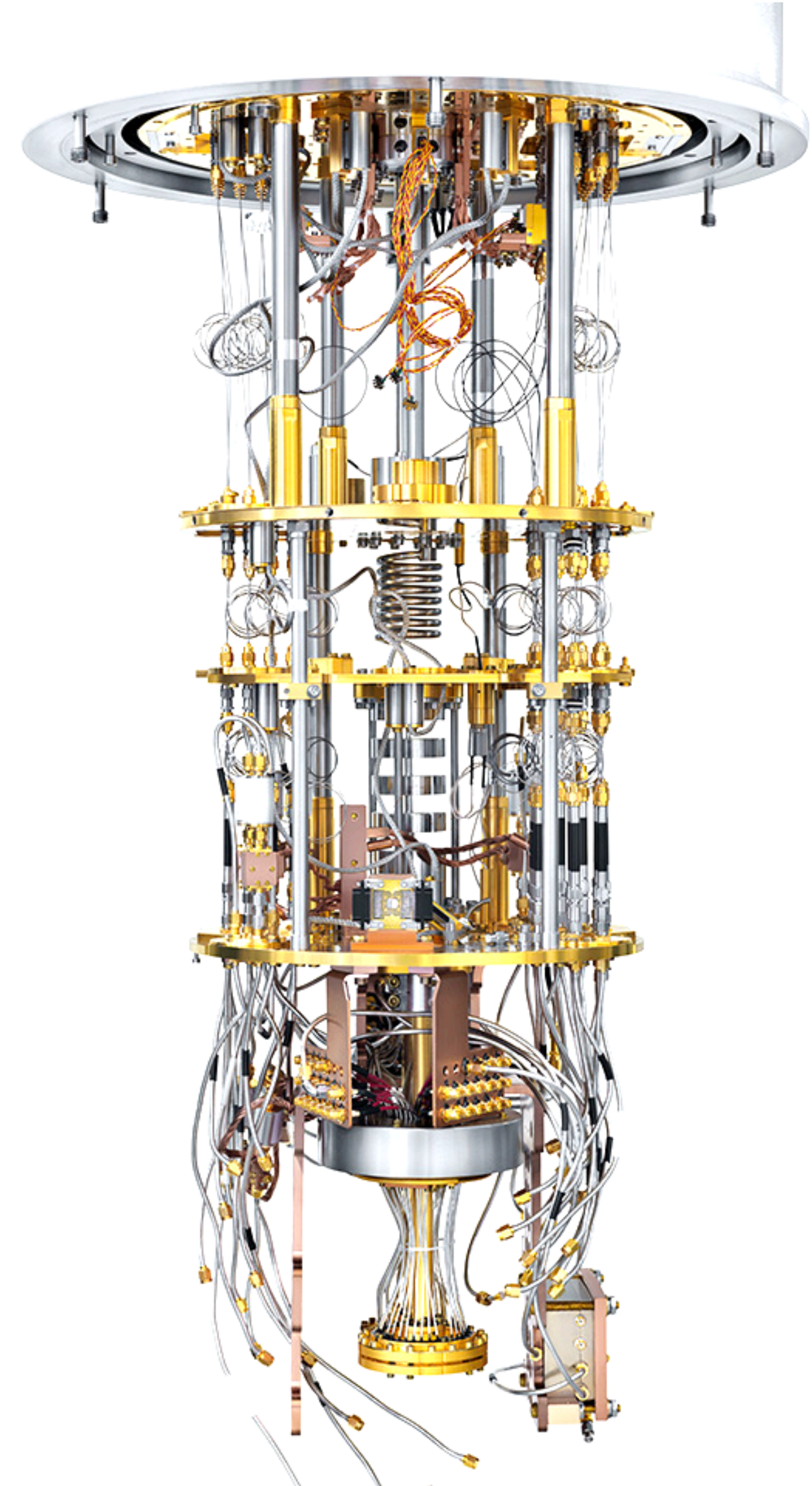
**Jump to the code**

# Next up

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**More sophisticated models**

**Barren plateaus**



# Thank you!

## NITheP Mini-school

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