The dynamics of quantum mechanical systems

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Online Workshop on
"Stochastic Dynamics of Quantum Mechanical Systems", 14 October 2021

60th anniversary of the paper on dynamical maps by

E. C. G. Sudarshan, P. M. Mathews and J. Rau

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Stochastic Dynamics of Quantum-Mechanical Systems

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AND

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(Received August 15, 1960)

The most general dynamical law for a quantum mechanical system with a finite number of levels is formulated. A fundamental role is played by the so-called "dynamical matrix" whose properties are stated in a sequence of theorems. A necessary and sufficient criterion for distinguishing dynamical matrices corresponding to a Hamiltonian time-dependence is formulated. The non-Hamiltonian case is discussed in detail and the application to paramagnetic relaxation is outlined.

George Sudarshan

September 16th, 1931, Pallam, Kerala, India.
May 13th, 2018, Austin, Texas, USA.

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Sudarshan: 7 SCIENCE QUESTS

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The Seven Quests

Professor George Sudarshan's work can be categorized into the following seven areas. The overview contains an introduction to the topic and the references link to a list of Professor Sudarshan's writings. All the links will open a page on the Friends of George Sudarshan web site in a new window. To return to this site, simply close the top window.

I **V-A: Universal Theory of Weak Interaction**
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II **Symmetry**
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III **Spin Statistics**
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V **Quantum Zeno Effect**
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VI **Theory of Tachyons**
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VII **Quantum Mechanics of Open Systems**
Overview | References

- An Invitation to Quantum Channels
  Vinayak Jagadish, Francesco Petruccione
  Quanta, Volume 7, Issue 1, Page 54 (July 2018)
- How the First Partial Transpose was Written
  Dagmar Bruß and Chiara Macchiavello
  Foundations of Physics, Vol. 35, No. 11, November 2005
- Geometry of Quantum States: An Introduction to Quantum Entanglement
  I. Bengtsson and K. Zyczkowski
  Cambridge Univ. Press, 2006 (2nd edition 2020)
- The Legacy of George Sudarshan
  G Marmo, S Pascazio
  Open Systems & Information Dynamics 26, 1950011 (2019)
- A brief history of the GKLS equation
  D Chruściński, S Pascazio
  Open Systems & Information Dynamics 24, 1740001 (2017)

closed Q systems

$$i\dot{\psi} = H\psi \quad \longleftrightarrow \quad \psi_t = U_t\psi_0$$

Schroedinger equation

open Q systems

$$\rho' = \Lambda\rho = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$$

Kraus representation

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A Brief History of the GKLS Equation

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Abstract. We reconstruct the chain of events, intuitions and ideas that led to the formulation of the Gorini, Kossakowski, Lindblad and Sudarshan equation.

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A Brief History of the GKLS Equation

Date	Event
1955	Stinespring publishes [42]
1961	Sudarshan, Mathews, and Rau publish [65]
1971	Kraus publishes [54]
1972	Kossakowski publishes [4]
26 March – 6 April 1973	Gorini attends Marburg conference, where Størmer and Kraus mention complete positivity
May 1974	Lindblad submits [58]
September – December 1974	Gorini and Kossakowski visit Sudarshan in Texas
December 1974	Lindblad visits Ingarden in Toruń
January 1975	Gorini visits Lindblad in Stockholm
1975	Choi publishes [57]
March and April 1975	GKLS articles [2, 3] are submitted
1980	Kraus spends sabbatical year at University of Texas at Austin; Sudarshan, Wheeler, A. Böhm and Wootters are there
1983	Kraus publishes [55]

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WHERE IS
COMPLETE
POSITIVITY?

7. The Mystery of Sudarshan-Mathews-Rau Paper

In 1961 Sudarshan, Mathews, and Rau [65] published a remarkable paper entitled *Stochastic Dynamics of Quantum Mechanical Systems.* In the abstract the authors state:

> *The most general dynamical law for a quantum mechanical system with a finite number of levels is formulated. A fundamental role is played by the so-called "dynamical matrix" whose properties are stated in a sequence of theorems. A necessary and sufficient criterion for distinguishing dynamical matrices corresponding to a Hamiltonian time-dependence is formulated.*

Sudarshan, Mathews, and Rau analyzed the evolution of the density matrix represented by the following linear relation

$$\rho_{rs}(t) = \sum_{r',s'} A_{rs,r's'}(t,t_0)\rho_{r's'}(t_0) \tag{36}$$

and found that $\rho_{rs}(t)$ defines a density matrix for $t > t_0$ if and only if the A matrix satisfies the following properties:

$$\begin{aligned} A_{sr,s'r'} &= \overline{A_{rs,r's'}}\,, && \text{(Hermiticity)} \\ \sum_{r,s,r',s'} \overline{x}_r x_s A_{rs,r's'} y_{r'} \overline{y_{s'}} &\geq 0\,, && \text{(positivity)} \\ \sum_r A_{rr,r's'} &= \delta_{r's'}\,. && \text{(trace-preservation)}\,. \end{aligned} \tag{37}$$

If one knows how to represent a matrix satisfying the above properties, the problem is solved. However, as the authors remarked, these conditions are *fairly complicated.* In order to solve the problem they proposed to analyze the matrix B defined by

$$B_{rr',ss'} := A_{rs,r's'}. \tag{38}$$

In modern language, B, which was named *dynamical matrix* in [65], is nothing but the realignment of A (see e.g. [66]). Now, the authors claimed (without

$$B_{rr',ss'} = \overline{B_{ss',rr'}}\,, \qquad \text{(Hermiticity)}$$

$$\sum_{r,s,r',s'} \overline{z}_{rr'} B_{rr',ss'} z_{ss'} \;\geq\; 0\,, \qquad \text{(positivity)}$$

$$\sum_{r} B_{rr',rs'} = \delta_{r's'}\,, \qquad \text{(trace-preservation)}$$

$$\sum_{r,s,r',s'} \overline{x}_r x_s B_{rr',ss'} y_{r'} \overline{y_{s'}} \;\geq\; 0$$

Not necessarily positive, but only block-positive

Darek Chruscinski 's and my opinion

This "error," anticipated the Kraus representation of a quantum channel.

It is clear that the remarkable SMR paper was written too early and the community in 1961 was not prepared to grasp its elegance and predictive power!

Markovian limit: GKLS equation

$$\mathcal{L}\rho \;=\; -i[H,\rho] + \frac{1}{2}\sum_j (2V_j\rho V_j^* - V_j^* V_j\rho - \rho V_j^* V_j)$$

The problem

- There is no problem
- But one should be aware that SMR first and GKLS later were looking for a "characterization"
- Nowadays this seems obvious
- But at those times it was not even obvious that a characterisation of the dynamics of open systems was necessary

closed Q systems

$$i\dot{\psi} = H\psi \quad \longleftrightarrow \quad \psi_t = U_t\psi_0$$

Schroedinger equation

open Q systems

$$\rho' = \Lambda\rho = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$$

Kraus-Sudarshan (or KSMR) representation

The dynamics of quantum mechanical systems

Very prolific ideas

- CPTP maps
- Entanglement
- Partial Transposition
- Time reversal
- Concept of “isolated” (does it also mean to be correlation free?)

Applied quantum Zeno effect at a Solvay conference: the speaker (at the center) cannot move, as he is being "closely observed" by Misra and Sudarshan

3. Anecdotes

George was endowed with a razor-sharp wit and a brilliant personality. His comments would often come out of the blue and leave his interlocutors stunned. We like to remember some of his jokes and comments. There are far too many, so we will opt for a personal choice.

In 1999 P. Facchi and S. Pascazio went to Naples, where G. Marmo was hosting George and Vittorio Gorini. The discussion hinged on the conditions that would yield the quantum Zeno effect. At the blackboard, someone said: the Hamiltonian is self-adjoint. After a split second George replied: why should a Hamiltonian be self-adjoint? Are symmetric Hamiltonians not sufficient? An article was born after that discussion [24], but the question still resounds in our memory. George would never take a postulate for granted.

A few years later, always in Naples (George was a regular visitor in Italy), while eating a delicious Neapolitan pizza, some of us were having an animated discussion on quantum mechanics and the projection postulate. George would listen, silent. At some point he said: it is a good thing that quantum mechanics does not depend on its foundations [25].

Years ago, George and his wife Bhamathi were flying from India back to Texas. Their flight was badly delayed and they missed their connecting flight, somewhere in the US. Bhamathi and George were exhausted after the

On 3 July 2014 George was invited to give a public talk at the Physics Department of the University of Bari. The lecture was on Weak Interactions and the auditorium was full. There were many questions at the end of the seminar. Someone (either a student or a postdoc) asked George an opinion about the discovery of the Higgs particle (that had been officially announced at CERN two years before). George said: young man, I would not worry about Higgs, I would rather ask: why the muon?

Bhamathi and George in Bari, Italy

Thank you