Quantum jumps and rate operators in open quantum system dynamics

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Collaboration: M. Caiaffa, D. Chruscinski, A. Smirne

Contents

1. Markovian: Monte Carlo Wave Function

Dalibard, Castin, Molmer
PRL 1992

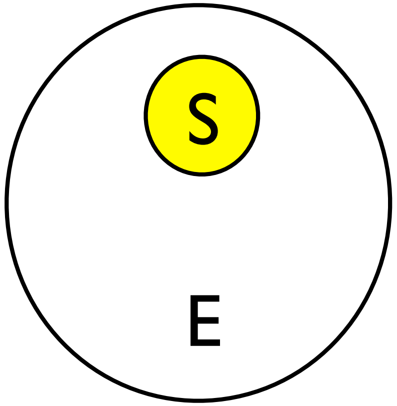
2. Non-Markovian Quantum Jumps

Piilo, Maniscalco, Härkönen, Suominen:
PRL 2008

3. Unified framework:
ROQJ - Rate operator quantum jumps

Smirne, Caiaffa, Piilo
PRL 2020
Chruscinski, Luoma, Piilo, Smirne
arXiv:2009.11312

Preliminaries: problem setting



- Any realistic quantum system S is coupled to its environment E
- Master equation description:

$$\frac{d\rho(t)}{dt} = -i[H, \rho_S] + \sum_k \gamma_k(t) \left(A_k \rho_S(t) A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S(t) - \frac{1}{2} \rho_S A_k^\dagger A_k \right)$$

- Decomposition of the density matrix

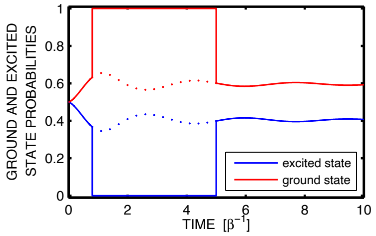
$$\rho(t) = \sum_i P_i(t) |\psi_i(t)\rangle\langle\psi_i(t)|$$

→ Stochastic descriptions

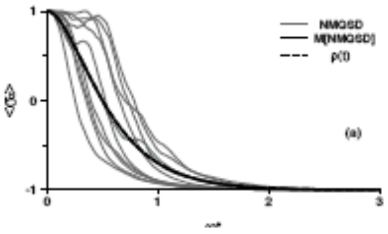
Simple classification of Monte Carlo/stochastic methods

	Markovian	non-Markovian
Jump methods:	MCWF (Dalibard, Castin, Molmer) Quantum Trajectories (Zoller, Carmichael)	Fictitious modes (Imamoglu) Pseudo modes (Garraway) Doubled H-space (Breuer, Petruccione) Triple H-space (Breuer) Non-Markovian Quantum Jump (Piilo et al)
Diffusion methods:	QSD (Diosi, Gisin, Percival...)	Non-Markovian QSD (Strunz, Diosi, Gisin, Yu) Stochastic Schrödinger equations (Barchielli)

Jump



Diffusion



Plus: Budini, Wiseman, Gambetta, Gaspard, Lacroix, Donvil and Muratore-Ginanneschi (not comprehensive list, apologies for any omissions)

Simple classification of Monte Carlo/stochastic methods

This talk

Markovian

non-Markovian

Jump methods:

Unified framework missing

moglu)

way)

euer, Petruccione)

r)

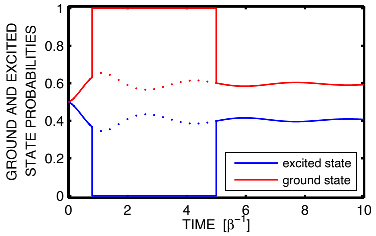
Non-Markovian Quantum Jump (Piilo et al)

Diffusion methods:

QSD
(Diosi, Gisin, Percival...)

Non-Markovian QSD
(Strunz, Diosi, Gisin, Yu)
Stochastic Schrödinger equations
(Barchielli)

Jump



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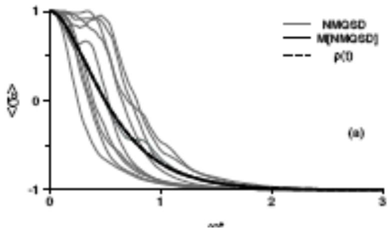

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Connection missing

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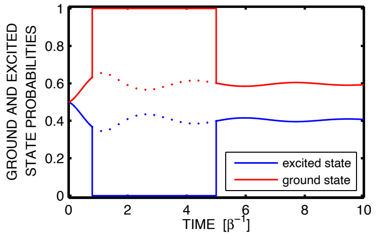

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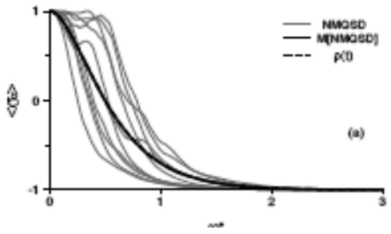

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For a solution:
see Luoma, Struntz, Piilo
PRL 2020

Markovian
Monte Carlo Wave Function method

Density matrix and state vector ensemble

Suppose now we want to solve the semigroup,
Markovian GKSL equation

$$\frac{d\rho(t)}{dt} = -i[H, \rho_S] + \sum_k \gamma_k \left(A_k \rho_S A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S - \frac{1}{2} \rho_S A_k^\dagger A_k \right)$$

Q: How to solve the master equation?

- Few exact models and analytical solutions
- Can we find the solution by evolving an ensemble of state vectors instead of directly solving the density matrix?

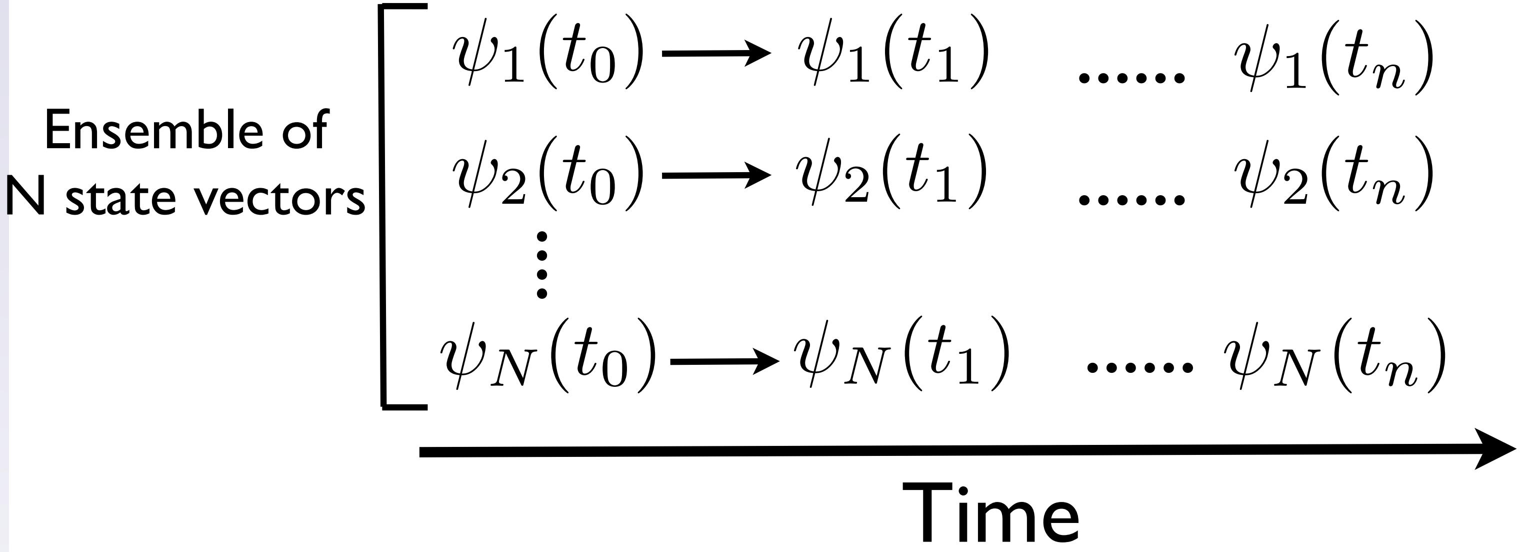
Generally, we can decompose the density matrix as

$$\rho(t) = \sum_i P_i(t) |\psi_i(t)\rangle\langle\psi_i(t)|$$

Basics of stochastic state vector evolution

Monte Carlo wave function method (Markovian)

(Dalibard, Castin, Molmer, PRL 1992)



Ensemble of N state vectors

$$\left[\begin{array}{llll} \psi_1(t_0) \longrightarrow & \psi_1(t_1) & \ldots\ldots & \psi_1(t_n) \\ \psi_2(t_0) \longrightarrow & \psi_2(t_1) & \ldots\ldots & \psi_2(t_n) \\ \vdots & & & \\ \psi_N(t_0) \longrightarrow & \psi_N(t_1) & \ldots\ldots & \psi_N(t_n) \end{array}\right.$$

Time

At each point of time, density matrix ρ as average of state vectors Ψ_i:

$$\rho(t) = \frac{1}{N} \sum_{i=1}^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

The time-evolution of each Ψ_i contains stochastic element due to random quantum jumps.

Jump probability, example

Time-evolution of state vector Ψ_i:

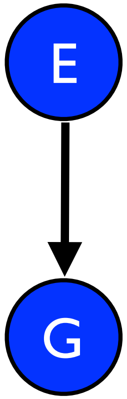
At each point of time: decide if quantum jump happened.

P_j: probability that a quantum jump occurs in a given time interval δt:

$$P_j = \underset{\text{time-step}}{\delta t}\;\underset{\text{decay rate}}{\Gamma}\;\underset{\text{occupation probability of excited state}}{p_e}$$

For example: 2-level atom

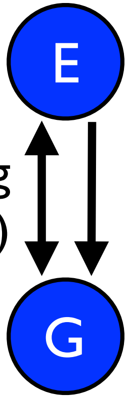
Probability for atom being transferred from the excited to the ground state and photon emitted.



Example: driven 2-state system, Markovian

Quantum jump: Discontinuous stochastic change of the state vector.

Excited state probability P
for a driven 2-level atom



Excited state

coupling
(deterministic)

decay channel
(random jump)

Ground state

$$\frac{d\varrho}{dt} = -i[H,\varrho] + \Gamma\left[\sigma_-\varrho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\varrho\}\right]$$

$$\rho(t) = \frac{1}{N}\sum_{i=1}^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

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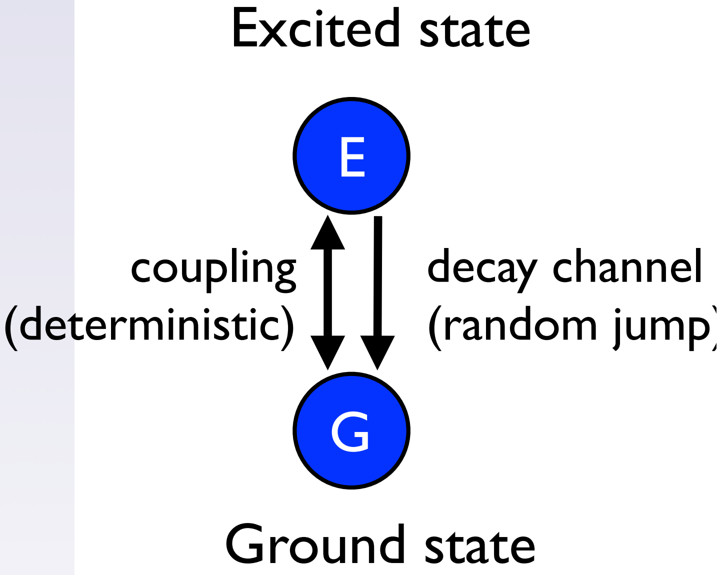


Excited state

E

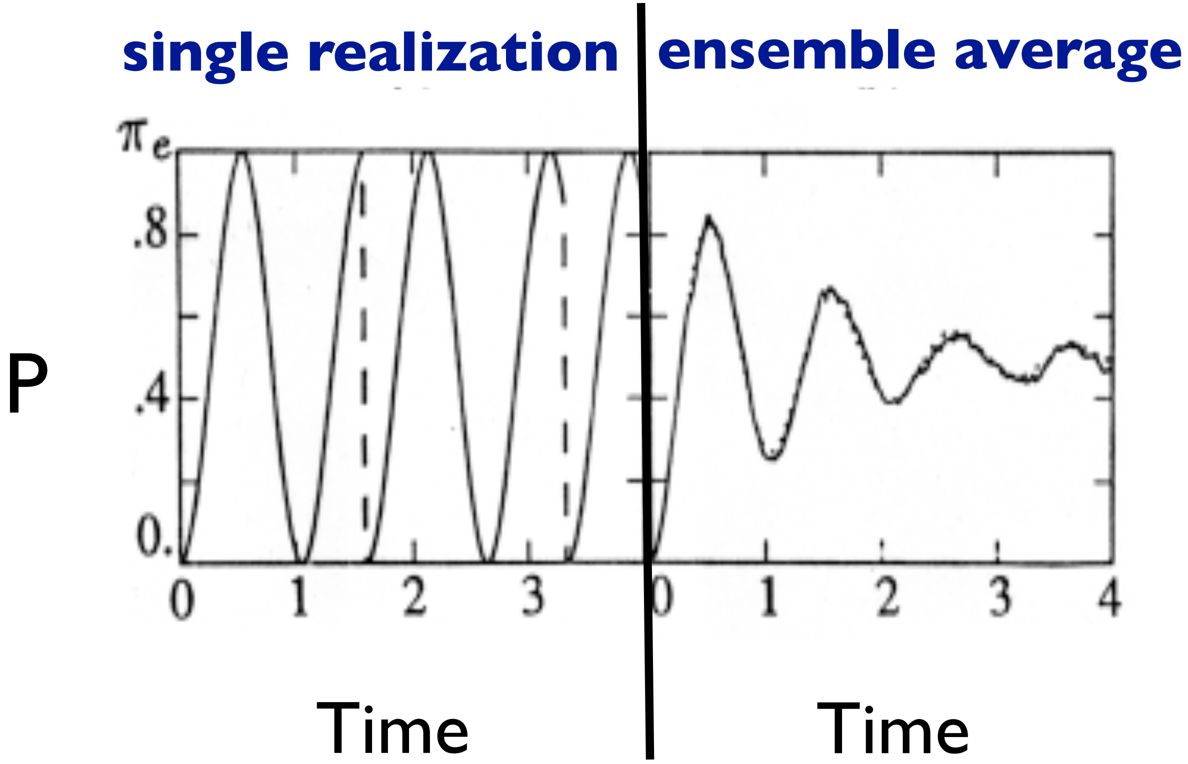
coupling
(deterministic)

decay channel
(random jump)

G

Ground state

Markovian Monte Carlo



single realization | **ensemble average**

damped Rabi oscillation
of the atom

$$\frac{d\varrho}{dt} = -i[H,\varrho] + \Gamma\left[\sigma_-\varrho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\varrho\}\right]$$

$$\rho(t) = \frac{1}{N}\sum_{i=1}^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

Markovian Monte Carlo wave function method

Master equation to be solved:

$$\boxed{\frac{d\rho(t)}{dt} = \frac{1}{i\hbar}\left[H_S,\rho\right] + \sum_m \Gamma_m C_m \rho C_m^{\dagger} - \frac{1}{2}\sum_m \Gamma_m \left(C_m^{\dagger} C_m \rho + \rho C_m^{\dagger} C_m\right)}$$

For each ensemble member ψ:

$$\boxed{i\hbar \frac{d}{dt}\left|\Psi(t)\right\rangle = H\left|\Psi(t)\right\rangle}$$

Solve the time dependent Schrödinger equation.

$$\boxed{H = H_s + H_{dec}}$$

Use non-Hermitian Hamiltonian H which includes the decay part $H_{dec.}$

$$\boxed{H_{dec} = -\frac{i\hbar}{2}\sum_m \Gamma_m C_m^{\dagger} C_m}$$

Key for non-Hermitian Hamiltonian: Jump operators C_m can be found from the dissipative part of the master equation.

$$\boxed{\delta p_m = \delta t \Gamma_m \left\langle \Psi \middle| C_m^{\dagger} C_m \middle| \Psi \right\rangle}$$

For each channel m the jump probability is given by the time step size, decay rate, and decaying state occupation probability.

Dalibard, Castin, Molmer: PRL 1992

Markovian Monte Carlo wave function method

Algorithm:

1. Time evolution over time step δt

2. Generate random number, did jump occur?

No

Yes

3. Renormalize ψ before new time step

$$|\psi_i(t+\delta t)\rangle = \frac{e^{-iH_{\text{eff}}\delta t}|\psi_i(t)\rangle}{\sqrt{1-\delta p}}$$

3. Apply jump operator C_j before new time step

$$|\psi_i(t+\delta t)\rangle = \frac{C_j|\psi_i(t)\rangle}{||C_j|\psi(t)\rangle||}$$

4. Ensemble average over ψ :s gives the density matrix and the expectation value of any operator A

$$\langle A\rangle(t) = \frac{1}{N}\sum_i \langle\psi_i(t)|A|\psi_i(t)\rangle$$

Questions:

- What happens when the decay rates depend on time?
- What happens when the decay rates turn temporarily negative?

$$\frac{d\rho(t)}{dt} = -i[H, \rho_S] + \sum_k \boxed{\gamma_k(t)} \left(A_k \rho_S(t) A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S(t) - \frac{1}{2} \rho_S A_k^\dagger A_k \right)$$

2.
Non-Markovian Quantum Jumps

Piilo, Maniscalco, Härkönen, Suominen:
PRL 2008

Markovian vs. non-Markovian evolution (1)

Markovian dynamics:
Decay rate constant
in time.

Non-Markovian dynamics:
Decay rate depends on time,
obtains temporarily negative values.

Markovian vs. non-Markovian evolution (1)

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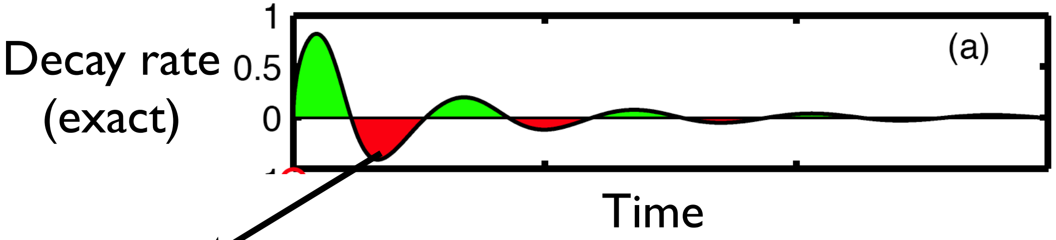
Example: 2-level atom in photonic band gap.



$$P_j = \delta t \, \Gamma \, p_e \ < 0$$

Markovian description of quantum jumps fails, since gives negative jump probability.
For example: negative probability that atom emits a photon.

Non-Markovian master equation

Starting point:

General non-Markovian master equation local-in-time:

$$\boxed{\frac{d\rho(t)}{dt}=\frac{1}{i\hbar}\left[H_S,\rho\right]+\sum_m \Delta_m(t)C_m\rho C_m^{\dagger}-\frac{1}{2}\sum_m \Delta_m(t)\left(C_m^{\dagger}C_m\rho+\rho C_m^{\dagger}C_m\right)}$$

- Jump operators C_m
- Time dependent decay rates $\Delta_m(t)$.
- Decay rates have temporarily negative values.

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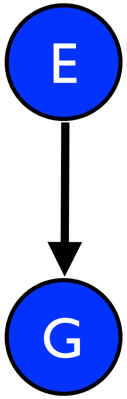
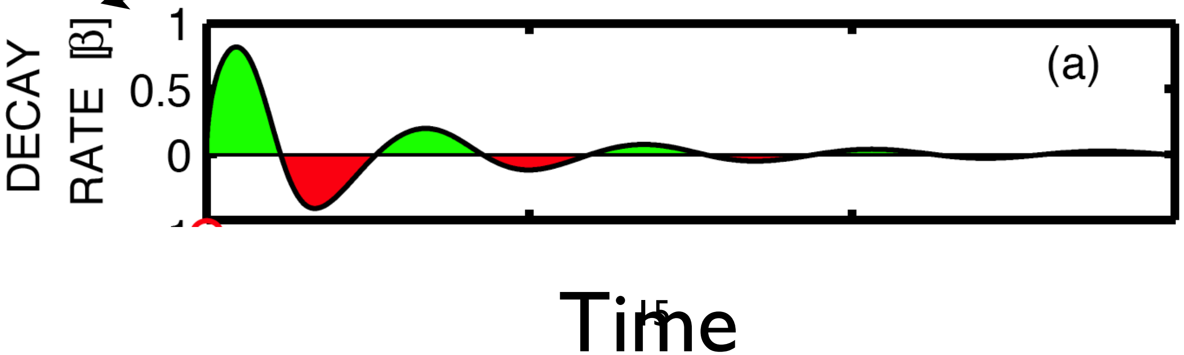


Example: 2-level atom in photonic band gap.

Jump operator C for positive decay: $\sigma_- = |g\rangle\langle e|$

$$\frac{d\rho(t)}{dt} = \frac{1}{i\hbar}[H_S,\rho] + \Gamma(t)|g\rangle\langle e|\rho|e\rangle\langle g| - \frac{1}{2}\Gamma(t)(|e\rangle\langle e|\rho + \rho|e\rangle\langle e|)$$



Non-Markovian quantum jump (NMQJ) method

Quantum jump in negative decay region:
The direction of the jump process reversed

$$|\psi\rangle \textcircled{\rightarrow} |\psi'\rangle = \frac{C_m|\psi\rangle}{||C_m|\psi\rangle||}, \quad \Delta_m(t) > 0$$

$$|\psi\rangle \textcircled{\leftarrow} |\psi'\rangle = \frac{C_m|\psi\rangle}{||C_m|\psi\rangle||}, \quad \Delta_m(t) < 0$$

Negative rate process creates coherences

Jump probability:

$$P = \frac{N}{N'}\delta t|\Delta_m(t)|\langle\psi|C_m^\dagger C_m|\psi(t)\rangle$$

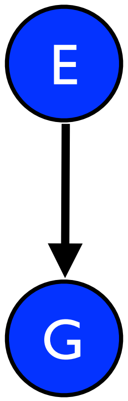
N: number of ensemble members in the target state
N': number of ensemble members in the source state

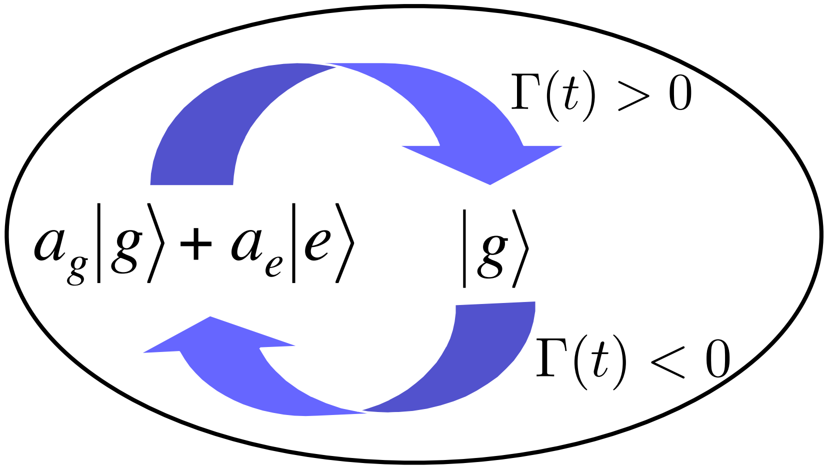
The probability proportional to the target state!

NMQJ example

For example: two-level atom

$$\sigma_- = |g\rangle\langle e|$$





$a_g|g\rangle + a_e|e\rangle$

$|g\rangle$

$\Gamma(t) > 0$

$\Gamma(t) < 0$

Jump probability: $P = \frac{N_0}{N_g} \delta t |\Gamma(t)| \; |\langle \psi_0 | e \rangle|^2$

The essential ingredient of non-Markovian system: memory. Recreation of lost superpositions.

NMQJ: general algorithm

$$\begin{aligned}\frac{d}{dt}\rho &= -i[H(t),\rho] \\ &+ \sum_k \Delta_k^+(t)\left[C_k(t)\rho C_k^\dagger(t) - \frac{1}{2}\left\{C_k^\dagger(t)C_k(t),\rho\right\}\right] \\ &- \sum_l \Delta_l^-(t)\left[C_l(t)\rho C_l^\dagger(t) - \frac{1}{2}\left\{C_l^\dagger(t)C_l(t),\rho\right\}\right]\end{aligned}$$

$$\rho(t) = \sum_\alpha \frac{N_\alpha(t)}{N}|\psi_\alpha(t)\rangle\langle\psi_\alpha(t)|$$ ensemble

Deterministic evolution and positive channel jumps as before...

Negative channel with jumps

$$D^{j-}_{\alpha\to\alpha'}(t) = |\psi_{\alpha'}(t)\rangle\langle\psi_\alpha(t)|$$

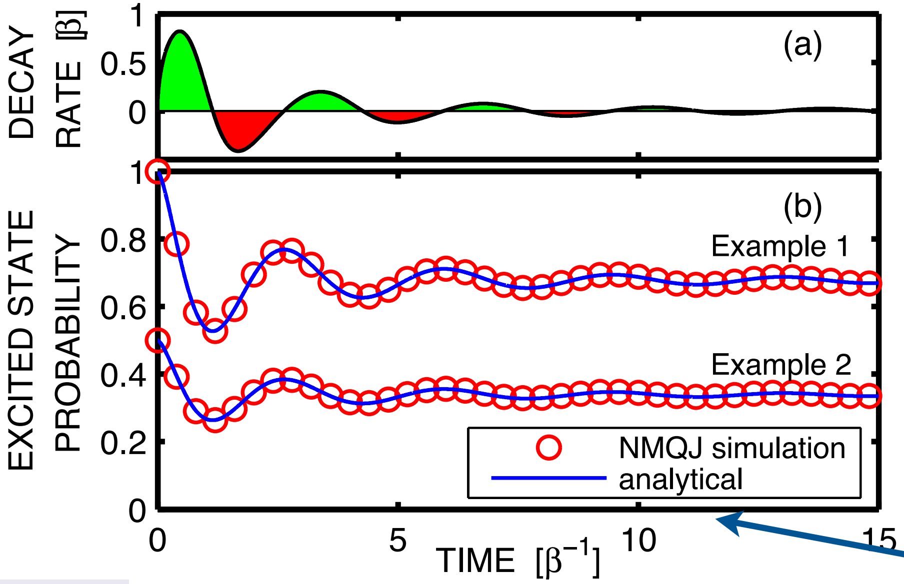
where the source state of the jump is

$$|\psi_\alpha(t)\rangle = C_{j_-}(t)|\psi_{\alpha'}(t)\rangle/||C_{j_-}(t)|\psi_{\alpha'}(t)\rangle||$$

...and jump probability for the corresponding channel

$$P^{j-}_{\alpha\to\alpha'}(t) = \frac{N_{\alpha'}(t)}{N_\alpha(t)}|\Delta_{j_-}(t)|\delta t\langle\psi_{\alpha'}(t)|C^\dagger_{j_-}(t)C_{j_-}(t)|\psi_{\alpha'}(t)\rangle.$$

Example: 2-level atom in photonic band gap



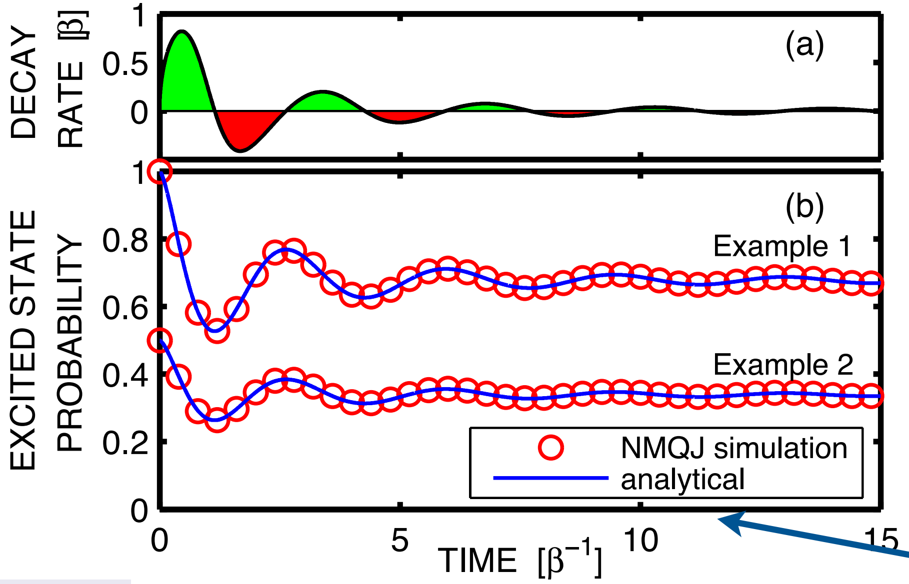
The simulation and exact results match.

Typical features of photonic band gap:

- Population trapping
- Atom-photon bound state.

Density matrix: average over the ensemble

Example: 2-level atom in photonic band gap



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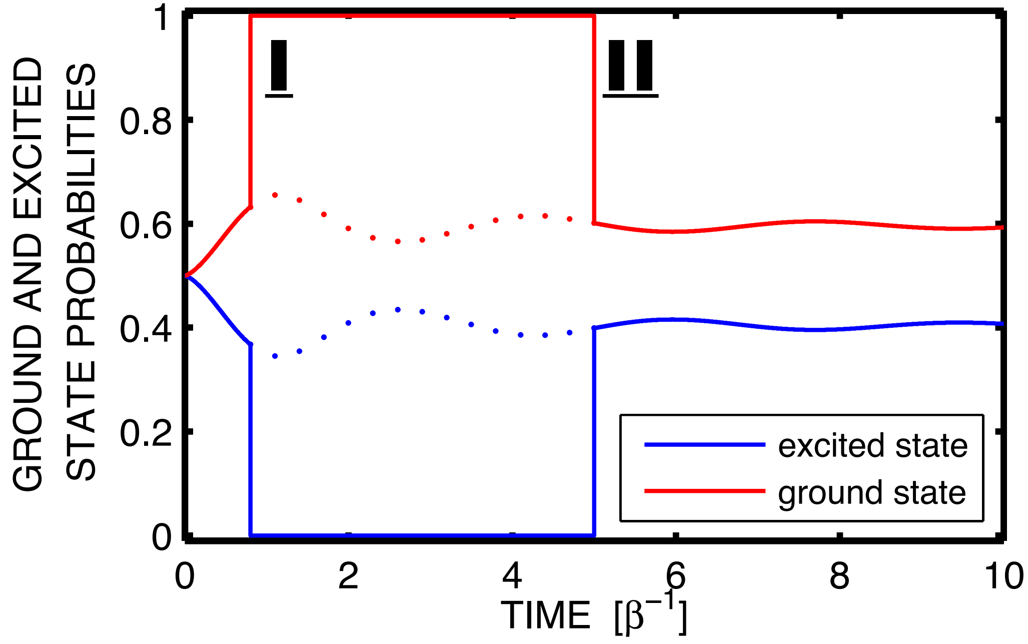
Density matrix: average over the ensemble

Single state vector history



Example of one state vector history:

I: Quantum jump at positive decay region destroys the superposition.

II: Due to memory, non-Markovian jump recreates the superposition.

Piilo, Maniscalco, Härkönen, Suominen: PRL 2008

3.
Unifying framework:
Rate operator quantum jumps (ROQJ)

Early/first consideration of ROs:
Diosi: Physics Letters 1986

On ROs for QSD:
Caiaffa, Smirne, Bassi:
PRA 2017

Smirne, Caiaffa, Piilo
PRL 2020

What is the problem? Example

"Eternal" non-Markovian master equation

$$\dot{\rho} = \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)[\sigma_k\rho\sigma_k - \rho],$$

Hall et al PRA 2014

- Pauli matrices σ_k
- Decoherence rates

$$\gamma_1(t) = \gamma_2(t) = 1, \quad \gamma_3(t) = -\tanh t \; < 0$$ for all $t > 0$

- Map CP but breaks CP-divisibility for all $t > 0$
- "Eternal" non-Markovian according to RHP criteria
- **...however, P-divisible for all** $t > 0$

“ENM” master equation

Why Markovian MCWF does not work?

- Rate for σ_z jump $\gamma_3(t) = -\tanh t$ <0 for all t>0

→ gives negative jump probability $P_j = \delta t \; \Gamma \; p_e \quad < 0$

Why non-Markovian NMQJ does not work?

- Reverse jump probility $P^{j-}_{\alpha \to \alpha'}(t) = \boxed{\frac{N_{\alpha'}(t)}{N_\alpha(t)}} |\Delta_{j_-}(t)| \delta t \langle \psi_{\alpha'}(t) | C^\dagger_{j_-}(t) C_{j_-}(t) | \psi_{\alpha'}(t) \rangle.$

→ singularity in the jump probability
(can not cancel something which never happened)

Note: however, fully classical Markovian description with ancillas exists

Megier, Chruscinski, Piilo, Strunz. Scientific Reports 2017

The problem and the motivation - once again…

- Processes exists which always break CP-divisibility and always preserve P-divisibility
- “In-between” Markovian and non-Markovian
- No known jump descriptions - without ancillas- exists

What is the most general stochastic jump description valid in all regimes?

Reminder about maps $\Phi_{t,0} = \Phi_{t,s}\Phi_{s,0}$:
CP-divisibility: $\Phi_{t,s}$ is CP
P-divisibility: $\Phi_{t,s}$ is P

ROQJ - Rate operator quantum jumps

- Master equation

$$\mathscr{L}[\rho(t)] = -\frac{i}{\hbar}[H_s, \rho(t)] + \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \left(L_\alpha(t)\rho(t)L_\alpha(t)^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger(t)L_\alpha(t), \rho(t)\right\} \right)$$

- At this stage, consider P-divisible dynamics.

- Negative rates allowed, as long as **transition rate operator** positive semi-definite (non-negative eigenvalues) for any pure state $|\psi(t)\rangle$

$$W^J_{\psi(t)} = \sum_{\alpha=1}^{n^2-1} c_\alpha(t)(L_\alpha(t) - \ell_{\psi(t),\alpha}) \left|\psi(t)\right\rangle\left\langle\psi(t)\right| (L_\alpha(t) - \ell_{\psi(t),\alpha})^\dagger$$

$$\ell_{\psi(t),\alpha} = \left\langle\psi(t)\right| L_\alpha(t) \left|\psi(t)\right\rangle$$

$$H_{\psi(t)} = H_S(t) - \frac{i\hbar}{2} \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \times \left(L_\alpha^\dagger(t)L_\alpha(t) - 2\ell^*_{\psi(t),\alpha}L_\alpha(t) + |\ell_{\psi(t),\alpha}|^2 \right)$$

deterministic evolution

ROQJ

- We can diagonalize and write with eigenvalues

$$W^J_{\psi(t)} = \sum_{j=1}^{n-1} \lambda_j(t) \left|\varphi_{\psi(t),j}\right\rangle\left\langle\varphi_{\psi(t),j}\right|$$
$$= \sum_{j=1}^{n-1} V_{\psi(t),j} \left|\psi(t)\right\rangle\left\langle\psi(t)\right| V^\dagger_{\psi(t),j},$$

- Here defined

$$V_{\psi(t),j} = \sqrt{\lambda_j(t)} \left|\varphi_{\psi(t),j}\right\rangle\left\langle\psi(t)\right|$$

Transfers from current state to eigenstate of rate operator with a rate given by the eigenvalue

- Therefore deterministic evolution interrupted by jumps

$$|\psi(t)\rangle \rightarrow \frac{V_{\psi(t),j}\,|\psi(t)\rangle}{\|V_{\psi(t),j}\,|\psi(t)\rangle\,\|}$$

which occur with probability

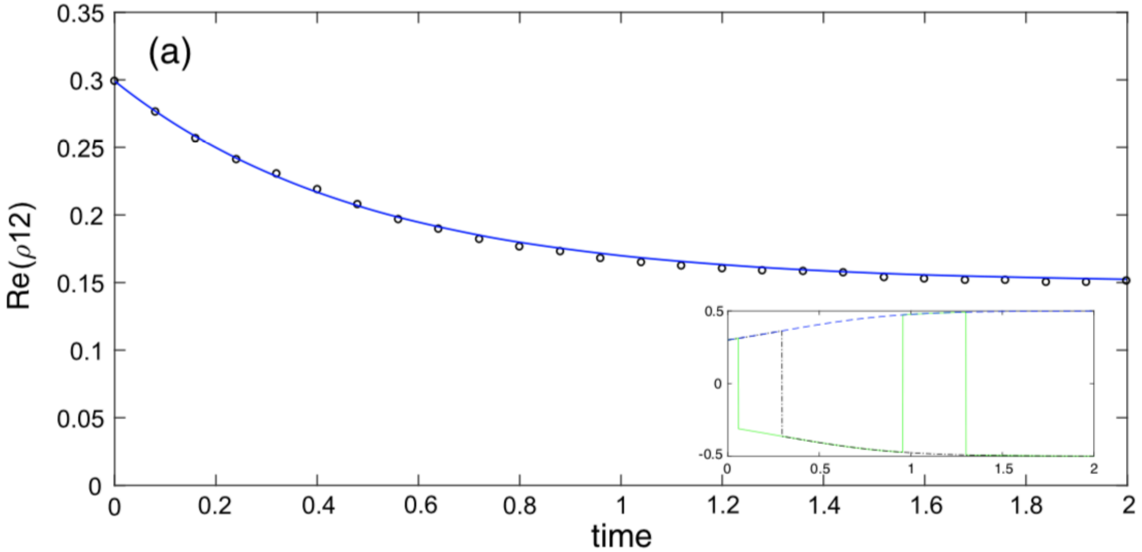
$$p_j(t) = \|V_{\psi(t),j}\,|\psi(t)\rangle\,\|^2 \mathrm{d}t.$$

- Similarity to MCWF.

ROQJ - example

"ENM" master equation

$$\dot{\rho} = \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)[\sigma_k\rho\sigma_k - \rho], \qquad \gamma_1(t) = \gamma_2(t) = 1, \quad \gamma_3(t) = -\tanh t,$$



Bloch vector components

1,2: $x_k(t) = \frac{1}{2}(1 + e^{-2t})x_k(0)$

3: $x_3(t) = e^{-2t}x_3(0),$

- Monotonic loss of coherence

- Simulation produces analytical results
- In general possible to prove match with master equation

Note on measurement scheme

- Markovian MCWP has measurement scheme interpretation
- No known measurement schemes in non-Markovian regime
  (Diosi PRL 2008; Gambetta, Wiseman PRL 2008)

Where is the border between the two? How do we lose measurement scheme interpretation?

ROQJ - measurement scheme

- It is possible to show in mathematically rigorous manner that the method has continuous measurement scheme interpretation following Barchielli and Belavkin JPhysA 1991
- Therefore measurement scheme exists for master equations with negative rates as long as P-divisible
- Operations for the count (jump) defined by

$$\mathcal{I}_{\omega_t,j}\rho = V_{\omega_t,j}\rho V^{\dagger}_{\omega_t,j}dt, \quad j = 1, \dots n,$$

$$V_{\psi(t),j} = \sqrt{\lambda_j(t)}\,|\varphi_{\psi(t),j}\rangle\langle\psi(t)|$$

- Trajectory upto time t

$$\omega_t = (t_1, j_1; t_2, j_2; \dots t_m, j_m)$$

- Corresponding state transformation

$$\rho \mapsto \frac{\mathcal{I}_{\omega_t,j}\rho}{\mathrm{Tr}\left\{\mathcal{I}_{\omega_t,j}\rho\right\}}$$

ROQJ - measurement scheme

- Operations for the count (jump) defined by

$$\mathcal{I}_{\omega_t,j}\rho = V_{\omega_t,j}\rho V^{\dagger}_{\omega_t,j}dt, \quad j = 1,\ldots n, \qquad V_{\psi(t),j} = \sqrt{\lambda_j(t)}\,|\varphi_{\psi(t),j}\rangle\langle\psi(t)|$$

Important points:

- The operations are conditioned on the whole trajectory
- For example: the past jumps influence what is the current state and may influence the diagonalization and the construction of the corresponding jump operator
- Measurement scheme: In addition of measurement record requires also computational resources and possibly time dependent basis for the measurement depending on the trajectory

ROQJ - general scheme for all regimes

General scheme including non-Markovian regime

- Divide transition rate operator to positive and negative eigenvalue parts

$$
\begin{aligned}
W^{J}_{\psi_k(t)} &= \sum_{j=1}^{n-1} \lambda_j(t) \left|\varphi_{\psi_k(t),j}\right\rangle\left\langle\varphi_{\psi_k(t),j}\right| \\
&= \sum_{j^+} \lambda_{j^+}(t) \left|\varphi_{\psi_k(t),j^+}\right\rangle\left\langle\varphi_{\psi_k(t),j^+}\right| - \sum_{j^-} \left|\lambda_{j^-}(t)\right| \left|\varphi_{\psi_k(t),j^-}\right\rangle\left\langle\varphi_{\psi_k(t),j^-}\right|
\end{aligned}
$$

- For positive part, use the earlier scheme

- For negative part, calculate the jump probabilities in similar manner as for NMQJ and use in the reverse jumps as a source the eigenstates of the transition rate operator

ROQJ - in non-P-div region

Reverse jumps, corresponding to negative eigenvalues of W, are

$$B_{\psi_k(t),\psi_{k'}(t),j-} = \sqrt{\left|\lambda_{\psi_{k'}(t),j-}\right|}\,|\psi_{k'}(t)\rangle\langle\psi_k(t)|$$

- Source of reverse jump is the eigenstate of W: $|\psi_k(t)\rangle = |\varphi_{\psi_{k'}(t),j-}\rangle$
- Probability given by $p_{j-}^{(k\to k')}(t) = \frac{N_{k'}(t)}{N_k(t)}\left|\lambda_{\psi_{k'}(t),j-}\right|\mathrm{d}t.$

One general framework for all regimes:

- When P-div: jumps to eig. states of rate operator W
- When P-div: broken: jumps out of the eig. states of W
- ROQJ works also when neither MCWF nor NMQJ works (when P-div. with negative rates)

ROQJ - in non-Markovian (non-P-div) region

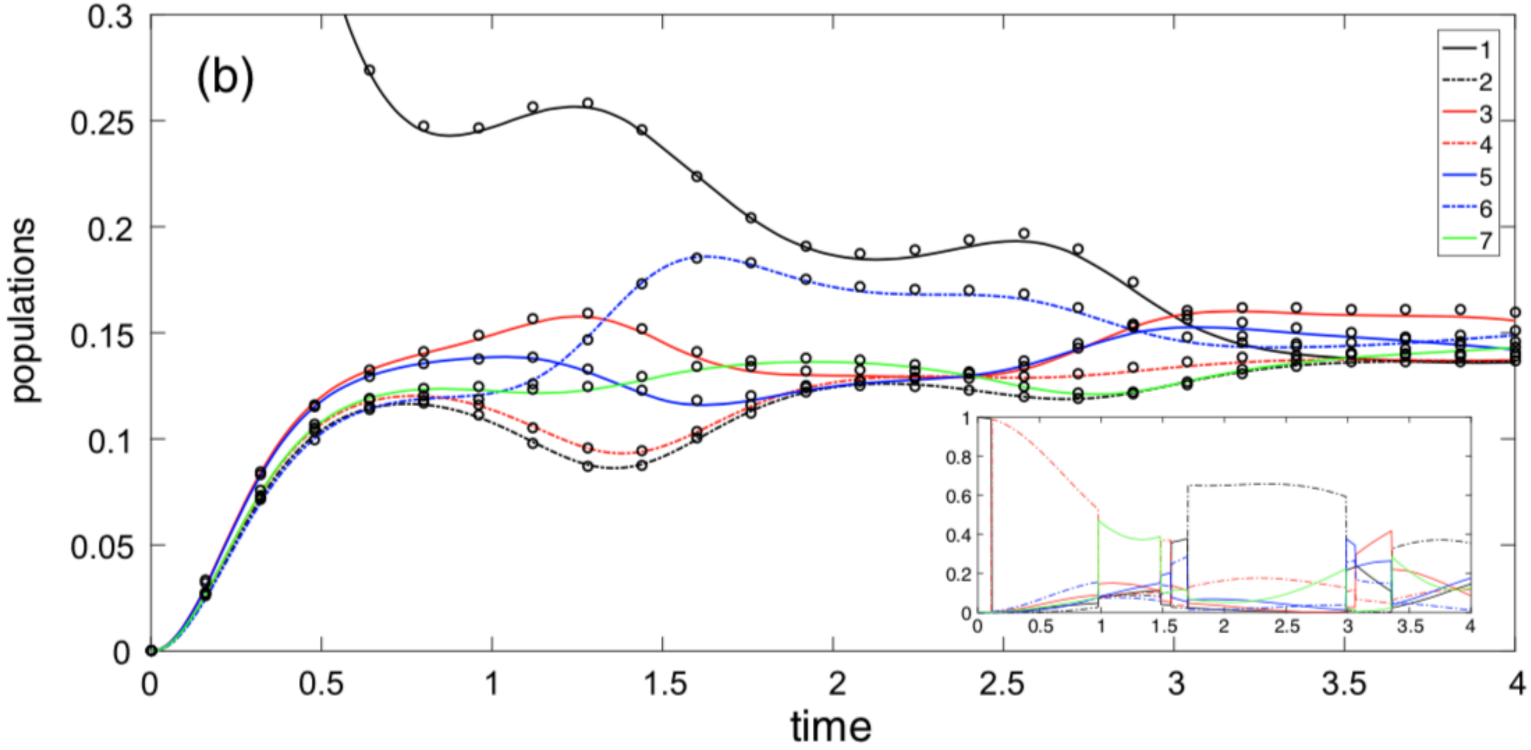
- 7-site driven system

  Unitary part: $H_S = \sum_{i \neq j} \Omega_{i,j} |i\rangle\langle j|$

  Jump (Lindblad) operators): $L_{i,j} = |i\rangle\langle j|$ (49 of them)

  Jump rates (contain negative regions):

  $c(t) = 0.5[(1 - e^{-0.5t})0.3 + e^{-0.3t} \sin(4.5t)]$



Is the rate operator unique?
Can we have a family of rate operators?

Chruscinski, Luoma, Piilo, Smirne
arXiv:2009:11312

Different choices for rate operator

- Master equation

$$\dot{\rho} = \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)[\sigma_k\rho\sigma_k - \rho],$$

For example

- Rate operator **R1**

$$\mathbf{R1} = \sum_{k=1}^{d}\gamma_k(t)\sigma_k|\psi\rangle\langle\psi|\sigma_k + \sum_k \gamma_k(t)|\psi\rangle\langle\psi|$$

$$K1(t) = \frac{i}{2}\gamma(t)1$$ deterministic evolution

- Rate operator **R2**

$$\mathbf{R2} = \sum_{k=1}^{d}\gamma_k(t)\sigma_k|\psi\rangle\langle\psi|\sigma_k + (\gamma_1(t) + \gamma_2(t))|\psi\rangle\langle\psi|$$

$$K2(t) = \frac{i}{2}[\gamma_1(t) + \gamma_2(t)]1$$ deterministic evolution

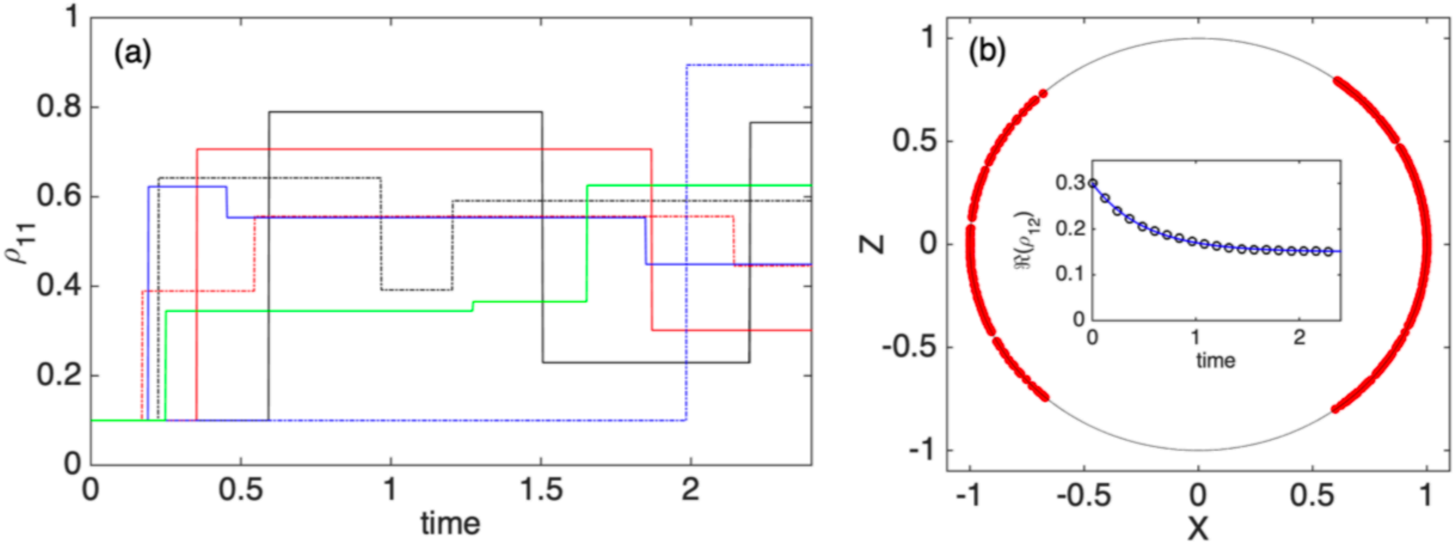
Chruscinski, Luoma, Piilo, Smirne
arXiv:2009:11312

Different choices for rate operator

With R1

Example realizations

Bloch vector x and z components: final distribution

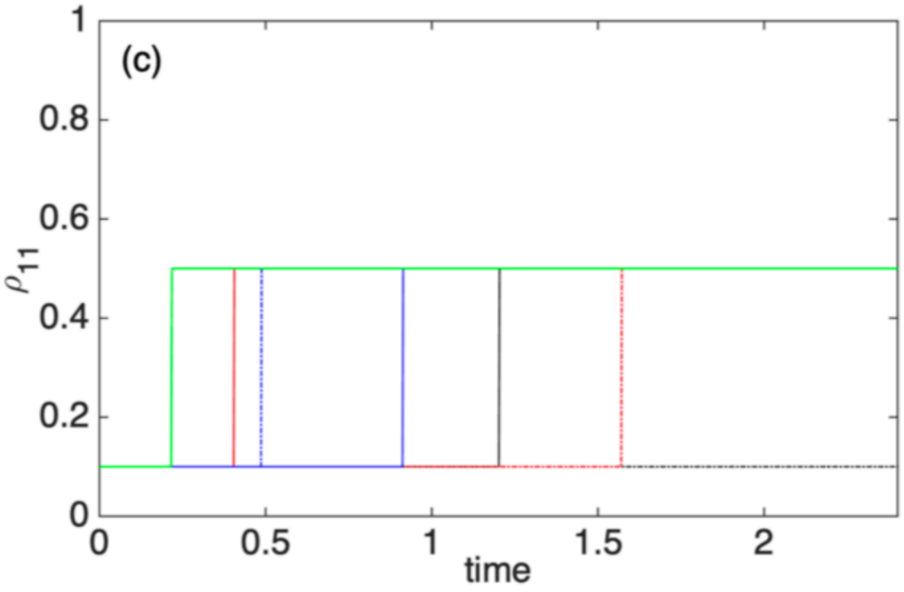


Chruscinski, Luoma, Piilo, Smirne
arXiv:2009:11312

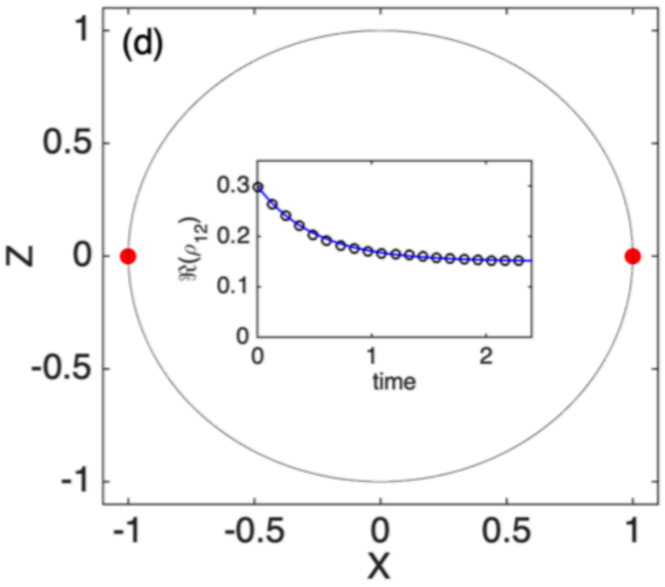
Different choices for rate operator

With R2

Example
realizations



Bloch vector x and z
components: final distribution



Chruscinski, Luoma, Piilo, Smirne
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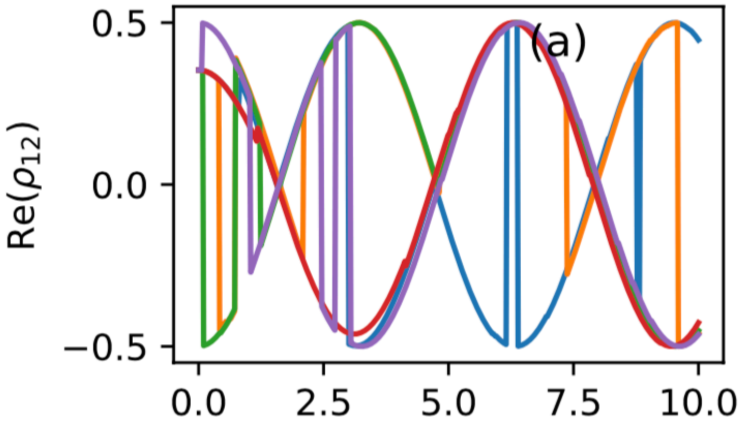
Different choices for rate operator

For unitarily driven system, we can also include the driving to jumps instead of deterministic evolution…

Example realizations:

R1: Driving in deterministic evolution:



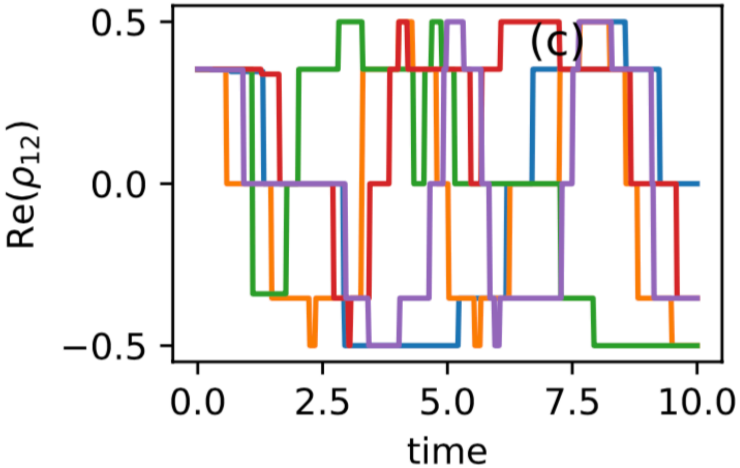
Dissipator:

$$\mathcal{L}_t(\rho) = \boxed{i\frac{b(t)}{2}[\sigma_z,\rho]} + \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)(\sigma_k\rho\sigma_k - \rho)$$

R1’: Driving included to jump (RO) part:

RO including driving:

$$\mathbf{R1'}_{\psi(t)} = \mathbf{R1}_{\psi(t)} \boxed{+\, i\frac{b(t)}{2}[\sigma_z, |\psi(t)\rangle\langle\psi(t)|]}$$



Chruscinski, Luoma, Piilo, Smirne
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CONCLUSIONS

- **MCWF** - Monte Carlo Wave Function (1992)
- **NMQJ** - Non-Markovian Quantum Jumps (2008)
- **ROQJ** - Rate Operator Quantum Jumps (2020)
  - General starting point for any regime
  - Unifies the framework for using quantum jumps to describe open system dynamics
  - Measurement scheme for master equations with negative rates (P-div, no ancillas used)
  - For the direction of having families of rate operators and engineering of trajectories…

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On non-uniqueness of the RO

Generator

$$\mathcal{L}_t(\rho) = -i[H(t),\rho] + \mathcal{J}_t(\rho) - \frac{1}{2}\{\Gamma(t),\rho\};$$

$$\mathcal{J}_t(\rho) = \sum_{\alpha=1}^{N^2-1} c_\alpha(t) L_\alpha(t)\rho L_\alpha^\dagger(t), \qquad \Gamma(t) = \mathcal{J}_t^\dagger(\mathbb{1}) = \sum_{\alpha=1}^{N^2-1} c_\alpha(t) L_\alpha^\dagger(t) L_\alpha(t)$$

We can also write

$$\mathcal{J}'_t(\rho) = \mathcal{J}_t(\rho) + \frac{1}{2}\left(\mathbf{C}(t)\rho + \rho\mathbf{C}^\dagger(t)\right)$$

$$\mathbf{C}(t) = A(t) + iB(t)$$

$$H'(t) = H(t) + \frac{1}{2}B(t), \quad \Gamma'(t) = \Gamma(t) + A(t)$$

which gives the same generator in the form

$$\mathcal{L}_t(\rho) = -i[H'(t),\rho] + \mathcal{J}'_t(\rho) - \frac{1}{2}\{\Gamma'(t),\rho\};$$