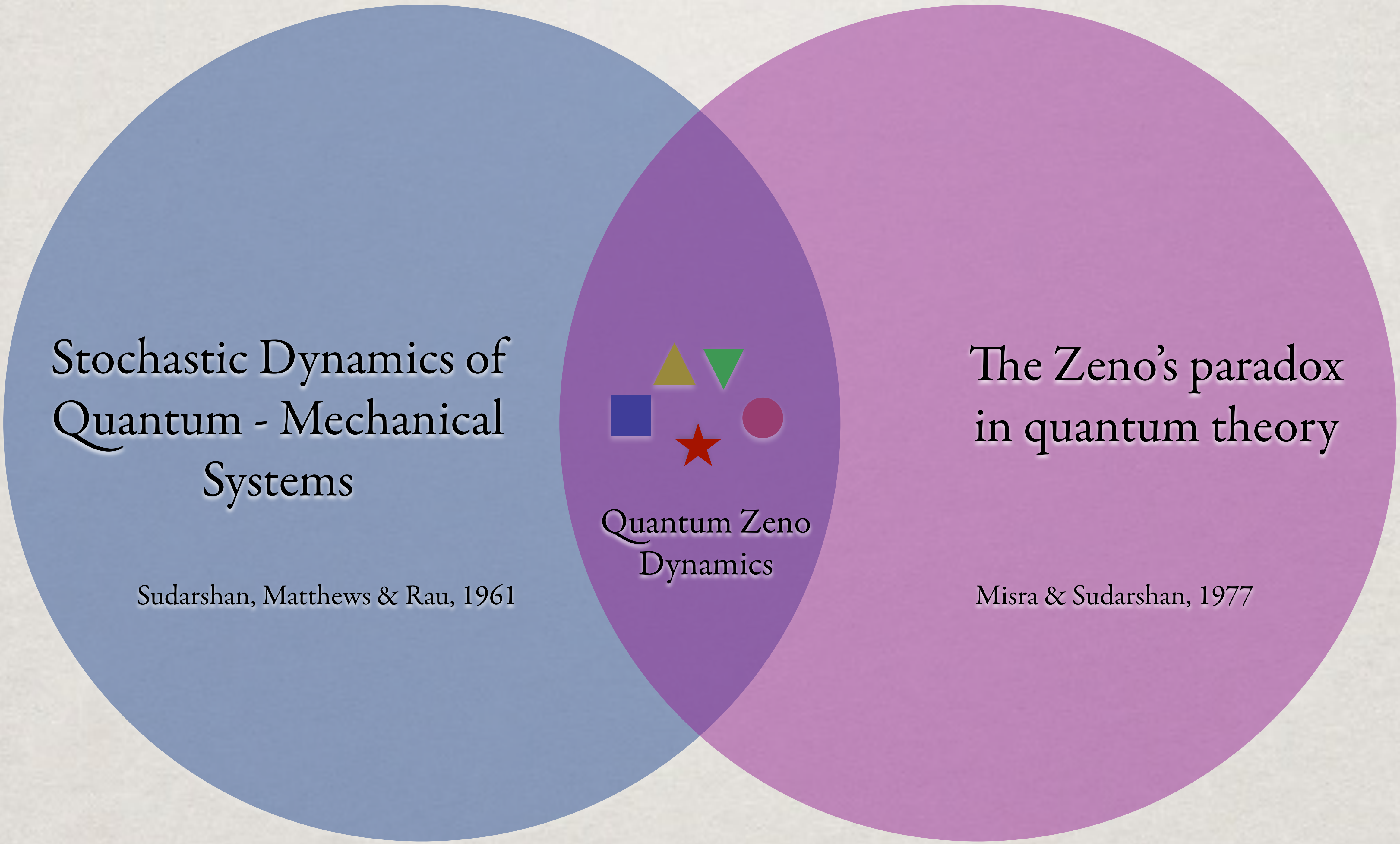
The Quantum Zeno Effect and its applications in Quantum Control

D. Burgarth, P. Facchi, H. Nakazato, S. Pascazio, K. Yuasa

@ Workshop Sudarshan, 14/10/2021

Overview


Stochastic Dynamics of Quantum - Mechanical Systems
Sudarshan, Matthews & Rau, 1961
Quantum Zeno Dynamics
The Zeno's paradox in quantum theory
Misra & Sudarshan, 1977

$\dim \mathcal{H} < \infty$

Quantum Zeno Effect (QZE)

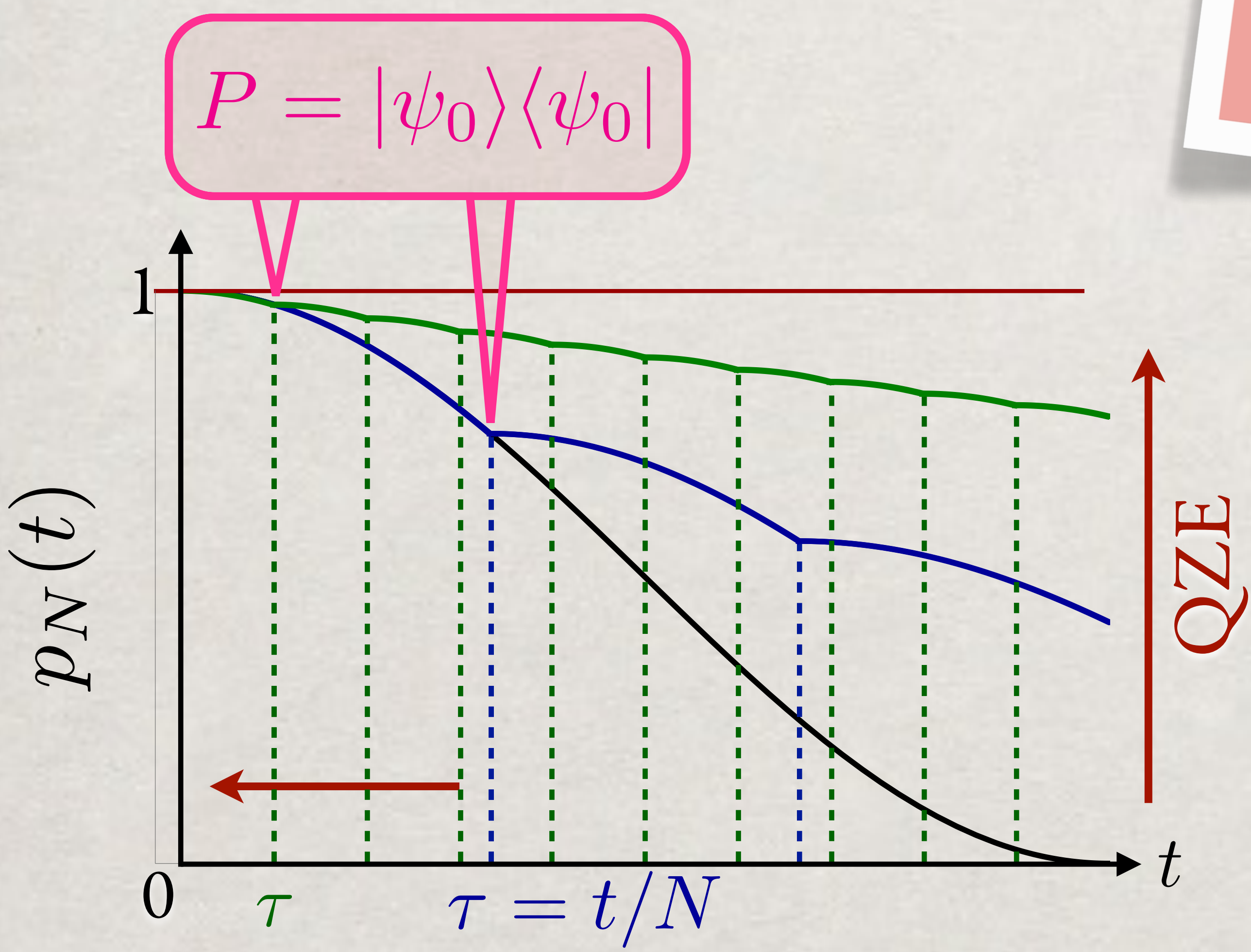
quadratic decay $\tau_Z^{-1} = \langle\psi_0|H^2|\psi_0\rangle - \langle\psi_0|H|\psi_0\rangle^2$

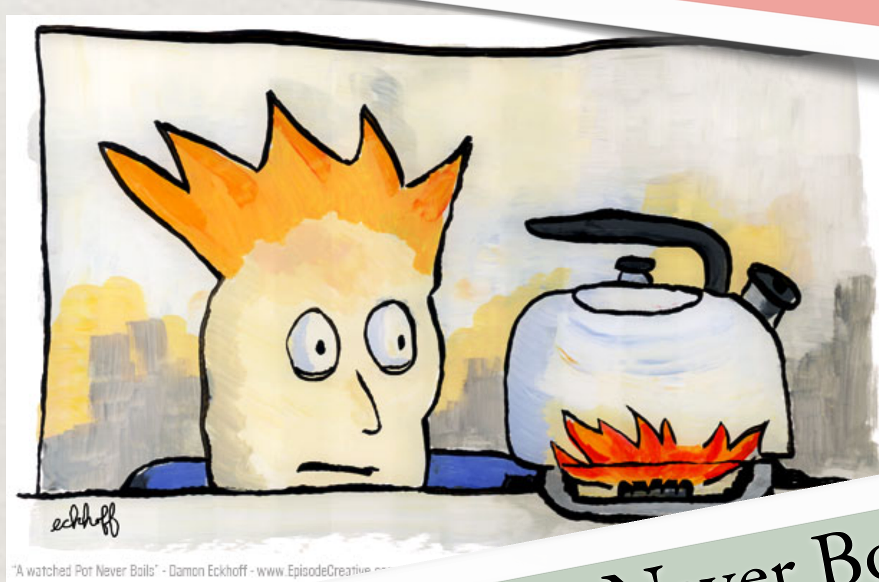
$$p(t) = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|^2$$

$$\sim \left|1 - i\langle\psi_0|H|\psi_0\rangle t - \frac{1}{2}\langle\psi_0|H^2|\psi_0\rangle t^2 + \cdots\right|^2 \sim 1 - \frac{t^2}{\tau_Z^2} + \cdots$$

Demonstrates "active" role of measurements in QM

rank one projection = full freeze





"A watched Pot Never Boils"

B. Misra and E. C. G. Sudarshan, "The Zeno's paradox in quantum theory," JMP **18**, 756 (1977);
P. Facchi and S. Pascazio, "Quantum Zeno dynamics: mathematical and physical aspects," JPA **41**, 493001 (2008).

Quantum Zeno Dynamics (QZD)

$$(Pe^{-iHt/N}P)^N$$

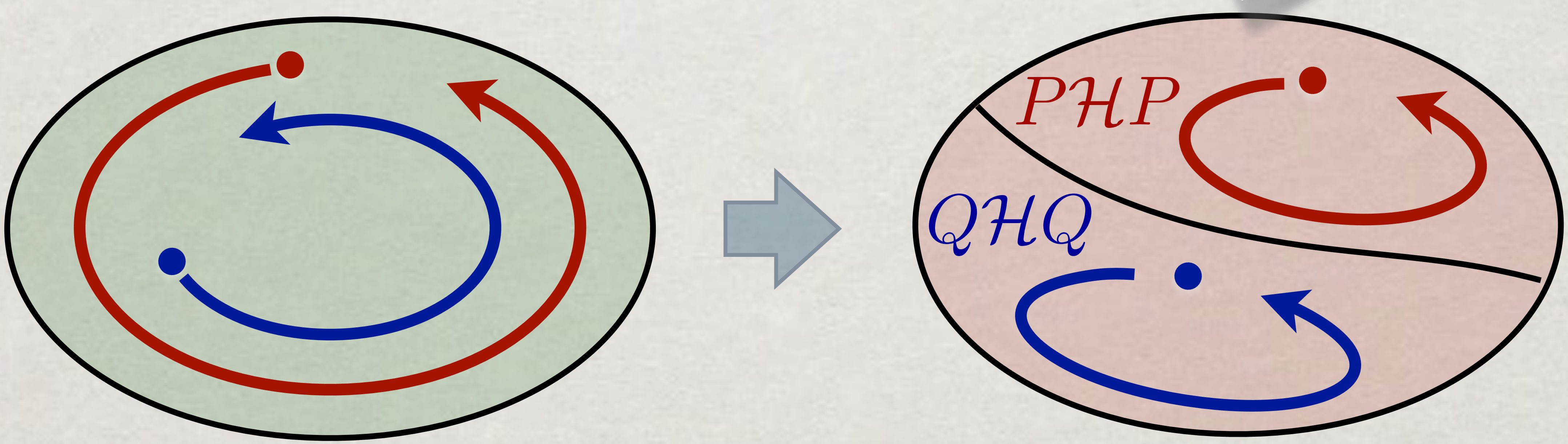
$$\sim \left[P\left(1 - iH\frac{t}{N} + \cdots\right)P\right]^N$$

$$\xrightarrow{N\to\infty} e^{-iPHPt}P$$

Zeno Hamiltonian

rank one: full freeze
full rank: nothing
in-between: QZD

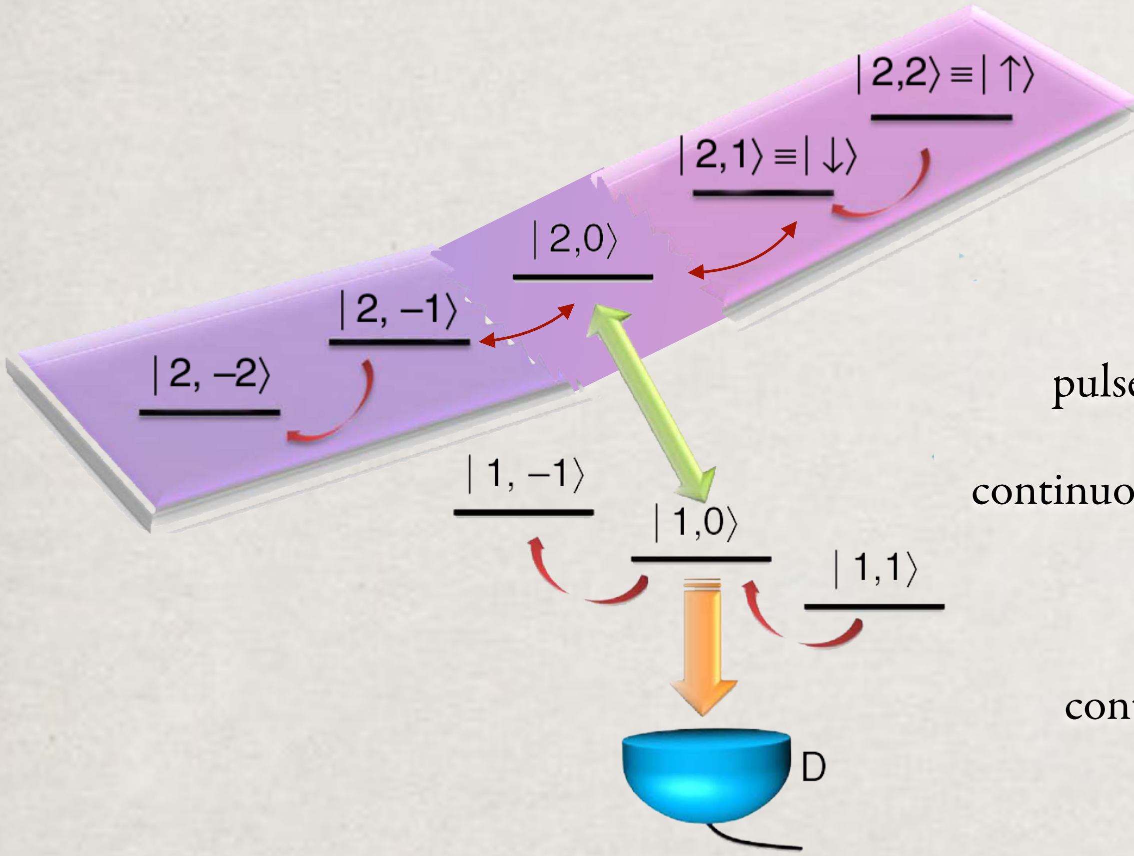
Valuable tool for Quantum Control



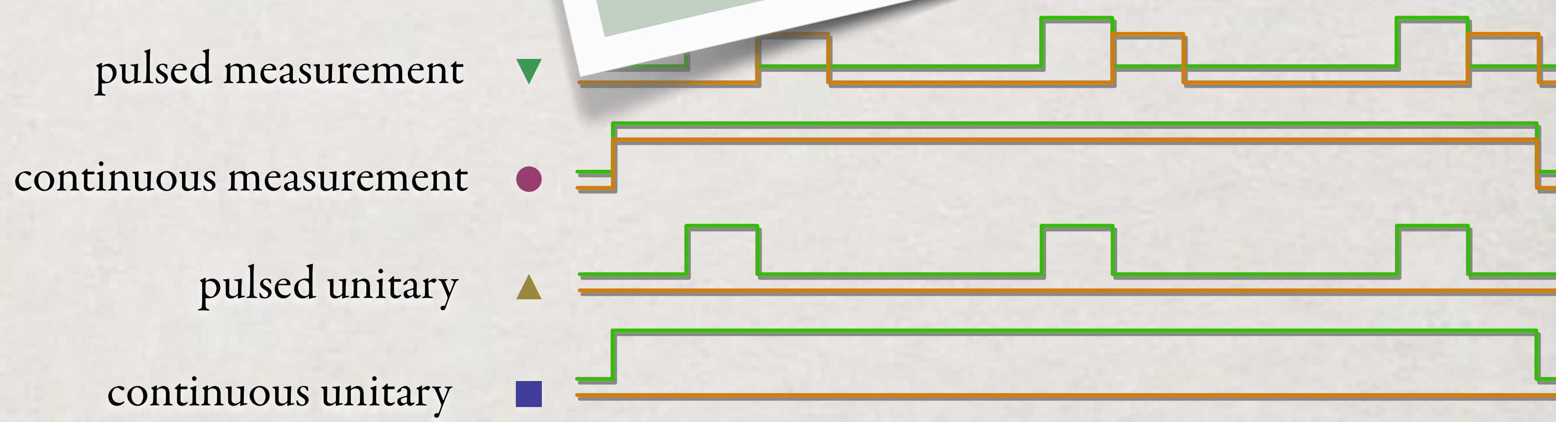
B. Misra and E. C. G. Sudarshan, "The Zeno's paradox in quantum theory," JMP **18**, 756 (1977);
P. Facchi and S. Pascazio, "Quantum Zeno subspaces," PRL **89**, 080401 (2002);
P. Facchi and S. Pascazio, "Quantum Zeno dynamics: mathematical and physical aspects," JPA **41**, 493001 (2008).

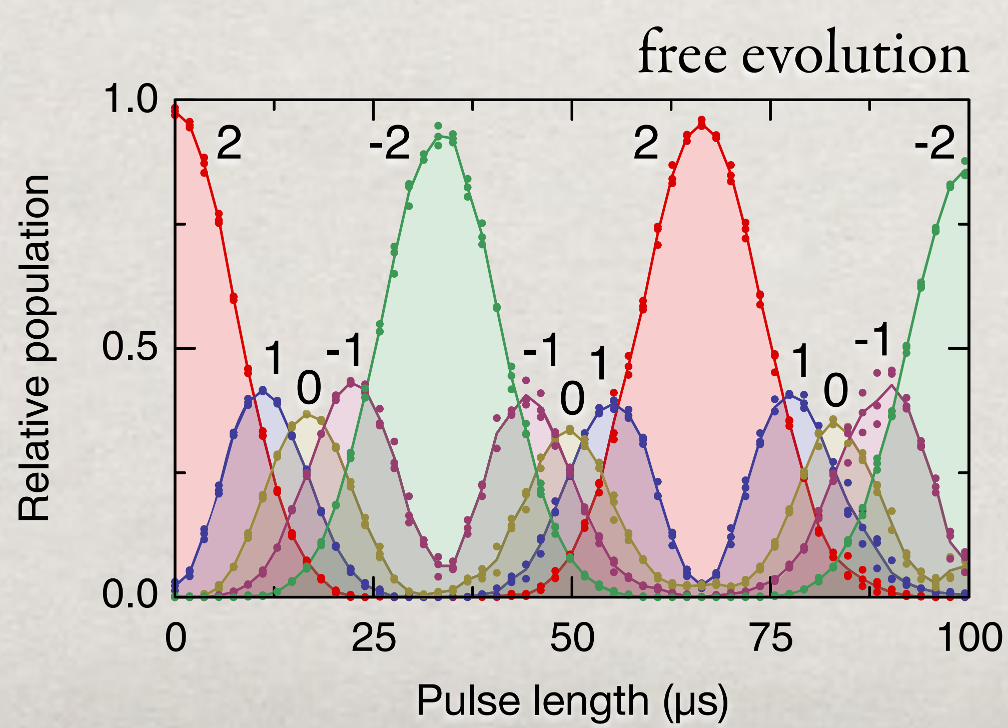
An Experimental Realization of QZE/QZD

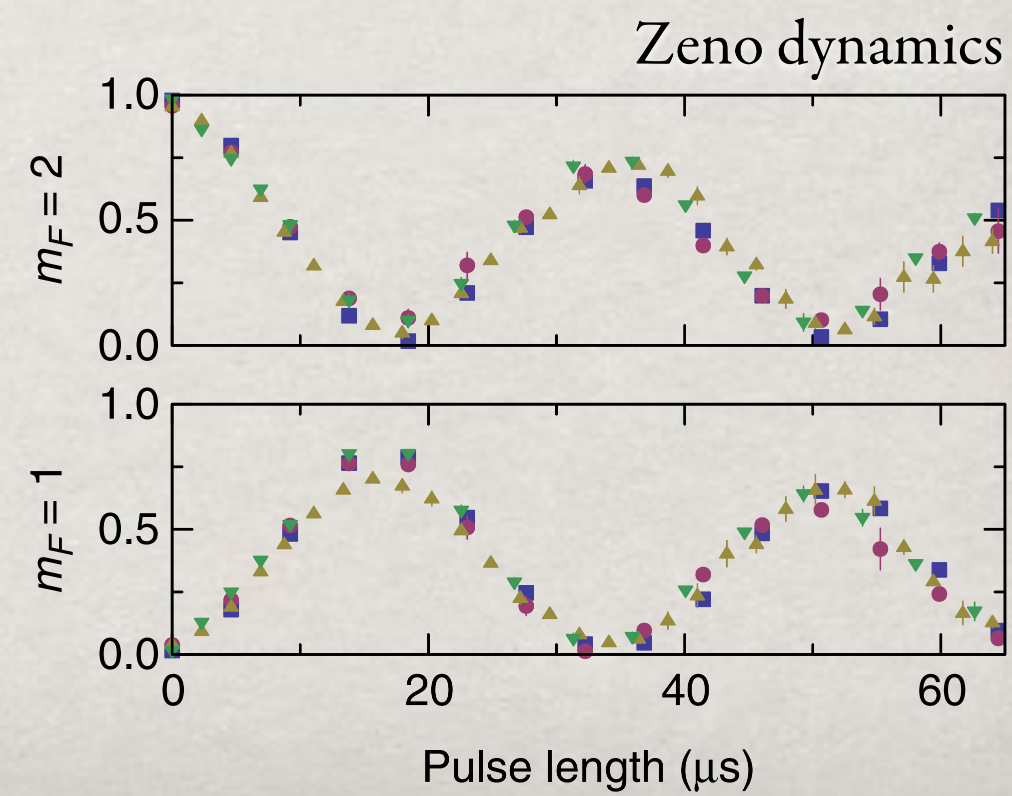
F. Schäfer *et al.*, "Experimental realization of quantum Zeno dynamics," Nat. Commun. **5**, 3194 (2014).



Different manifestations, same limit







Different Manifestations of QZE/QZD

- repeated measurements

$$(Pe^{-iHt/N})^N \xrightarrow{N\to\infty} e^{-iPHPt}P$$

B. Misra and E. C. G. Sudarshan, "The Zeno's paradox in quantum theory," JMP **18**, 756 (1977).

- dynamical decoupling / bang-bang control / repeated unitary kicks

$$(Ue^{-iHt/N})^N \xrightarrow{N\to\infty} U^N e^{-i\sum_n P_n H P_n t}$$

$$(U_m e^{-iHt/N}\cdots U_1 e^{-iHt/N})^N \xrightarrow{N\to\infty} (U_m\cdots U_1)^N e^{-i\sum_n P_n \overline{H} P_n t}$$

Generalize to CPTP maps

L. Viola and S. Lloyd, "Dynamical suppression of decoherence in two-state quantum systems," PRA **58**, 2733 (1998);
P. Facchi, D. A. Lidar, and S. Pascazio, "Unification of dynamical decoupling and the quantum Zeno effect," PRA **69**, 032314 (2004);
J. Z. Bernád, "Dynamical control of quantum systems in the context of mean ergodic theorems", JPA50, 065303 (2017).

- strong continuous field

$$e^{-i(\gamma H_0+H)t} \xrightarrow{\gamma\to\infty} e^{-i\gamma H_0 t} e^{-i\sum_n P_n H P_n t}$$

P. Facchi and S. Pascazio, "Quantum Zeno subspaces," PRL **89**, 080401 (2002).

strong damping/continuous measurement

$$e^{(\gamma\mathcal{L}_0+\mathcal{K})t} \xrightarrow{\gamma\to\infty} e^{\mathcal{P}_\varphi \mathcal{K}\mathcal{P}_\varphi t}\mathcal{P}_\varphi, \qquad e^{\gamma\mathcal{L}_0 t} \xrightarrow{\gamma\to\infty} \mathcal{P}_\varphi$$

Unify

P. Zanardi and L. Campos Venuti, "Coherent quantum dynamics in steady-state manifolds of strongly dissipative systems," PRL **113**, 240406 (2014);
V. V. Albert, B. Bradlyn, M. Fraas, L. Jiang, "Geometry and response of Lindbladians" PRX 6, 041031 (2016).

P. Facchi *et al.*, "Control of decoherence: analysis and comparison of three different strategies," PRA **71**, 022302 (2005);
P. Facchi and S. Pascazio, "Quantum Zeno dynamics: mathematical and physical aspects," JPA **41**, 493001 (2008).

Unified theory of Zeno through adiabatic theorem

strong damping
& strong field
■ ●

$$e^{(\gamma\mathcal{L}_0+\mathcal{L})t} = e^{\gamma\mathcal{L}_0 t} e^{\mathcal{L}_Z t}\mathcal{P}_\varphi + \mathcal{O}(1/\gamma)$$

$$\mathcal{L}_Z = \sum_{\alpha_k\in i\mathbb{R}} \mathcal{P}_k\mathcal{L}\mathcal{P}_k, \quad \mathcal{L}_0 = \sum_k(\alpha_k\mathcal{P}_k + \mathcal{N}_k), \quad \mathcal{P}_\varphi = \sum_{\alpha_k\in i\mathbb{R}} \mathcal{P}_k$$

measurements
unitary kicks
★ CPTP kicks
▼ ▲

$$(\mathcal{E}e^{\mathcal{L}t/n})^n = \mathcal{E}_\varphi^n e^{\mathcal{L}_Z t} + \mathcal{O}(1/n)$$

$$\mathcal{L}_Z = \sum_{|\lambda_k|=1} \mathcal{P}_k\mathcal{L}\mathcal{P}_k, \quad \mathcal{E} = \sum_k(\lambda_k\mathcal{P}_k + \mathcal{N}_k), \quad \mathcal{E}_\varphi = \sum_{|\lambda_k|=1} \lambda_k\mathcal{P}_k$$

▲ bang-bang
dynamical
decoupling
★ multi-CPTP
multi meas.

$$(\mathcal{E}_m e^{\mathcal{L}t/n}\cdots\mathcal{E}_2 e^{\mathcal{L}t/n}\mathcal{E}_1 e^{\mathcal{L}t/n})^n = \mathcal{E}_\varphi^n e^{\mathcal{L}_Z t} + \mathcal{O}(1/n)$$

$$\mathcal{L}_Z = \sum_{|\lambda_k|=1} \mathcal{P}_k\overline{\mathcal{L}}\mathcal{P}_k, \quad \overline{\mathcal{L}} = \mathcal{E}_\varphi^{-1}\sum_{i=1}^{m}\mathcal{E}_m\cdots\mathcal{E}_i\mathcal{L}\mathcal{E}_{i-1}\cdots\mathcal{E}_1$$

$$\mathcal{E} = \mathcal{E}_m\cdots\mathcal{E}_1 = \sum_k(\lambda_k\mathcal{P}_k + \mathcal{N}_k), \quad \mathcal{E}_\varphi = \sum_{|\lambda_k|=1} \lambda_k\mathcal{P}_k$$

D. Burgarth, K. Yuasa, P. Facchi, H. Nakazato, S. Pascazio, Quantum, 3/152 '19 & 4/298 '20

Non-commutativity at heart of control theory

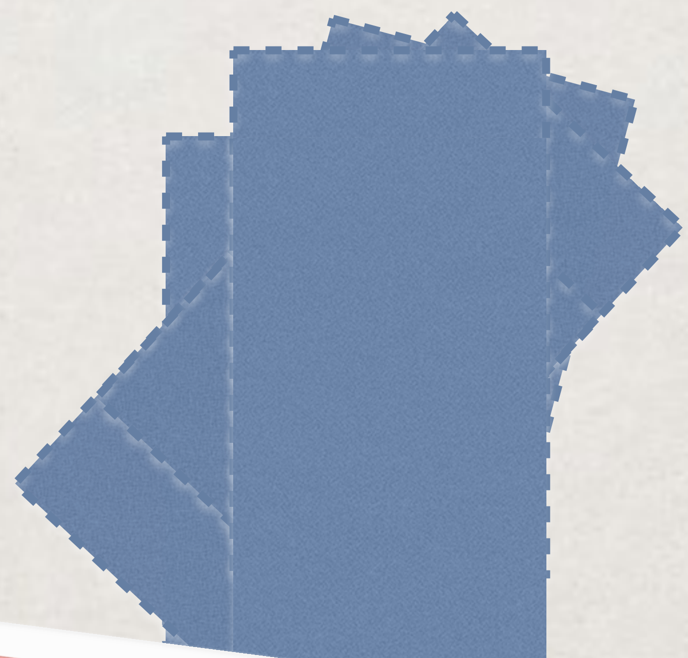
Commutative? { Steer, Drive }

Of course not!

[Steer, Drive] = Rotate

[Rotate, Drive] = Slide

Drive + Slide = Any direction



How does QZD affect commutativity?
If $[H_1, H_2] = 0$ do we have $[PH_1P, PH_2P] = 0$?

Lie Algebras, Lie Groups, Trotter theorems, Differential Geometry, ...

These techniques let us understand where we can drive in Hilbert space !

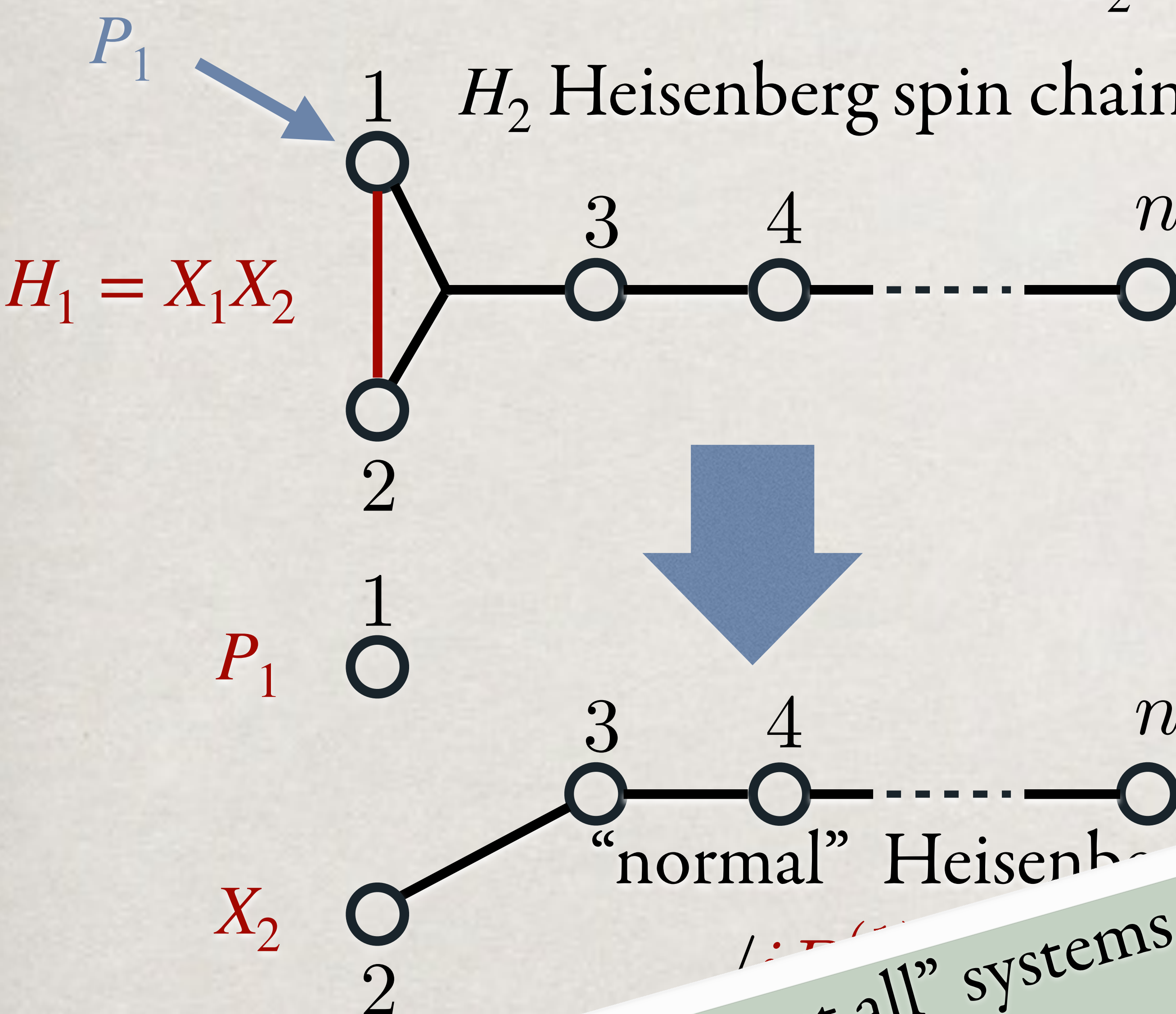
E. Nelson, *Tensor Analysis*, Princeton University Press, 1967

$\mathcal{H} = \mathbb{C}_2^{\otimes n}$

The Power of Zeno for Control

$$H_2 = \sum_{k=3}^{n-1} (XX + YY + ZZ)_{k,k+1} + Z_3 + H_3$$

$$H_3 = \sqrt{3}(XXX + YYY + ZZZ)_{123}$$



$[H_1, H_2] = 0$

$f(t)H_1 + g(t)H_2$ trivial dynamics

with single qubit projection

$$P_1 \sigma P_1 = \frac{1}{\sqrt{3}} \quad (\sigma = X, Y, Z)$$

pin chain

$^{(1)} H_2 P^{(1)}\rangle_{[\cdot,\cdot]} = P^{(1)} \otimes \mathrm{su}(2^{n-1})$

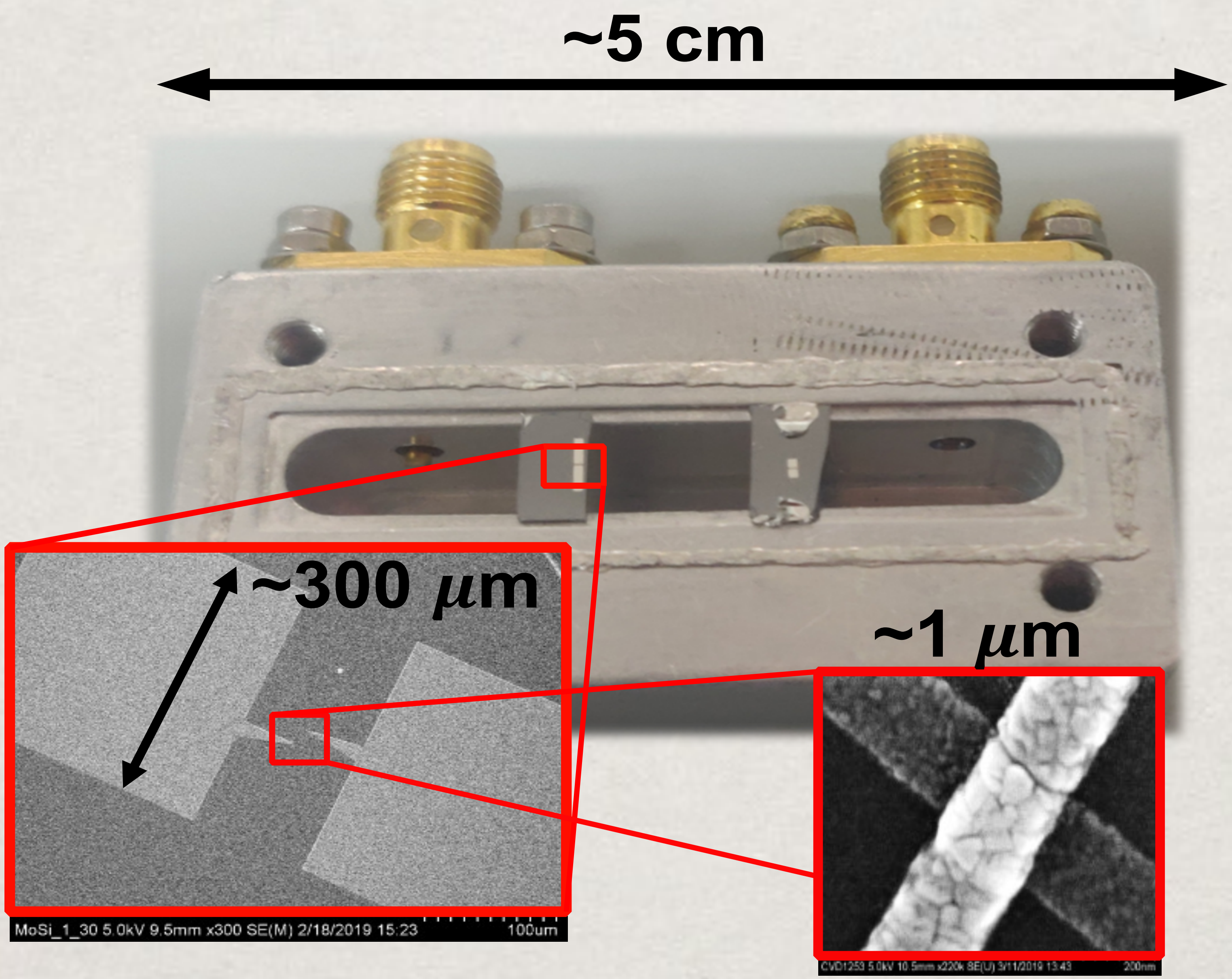
edge drivi

becomes universal for quantum computing

This happens for “almost all” systems, even noisy ones.

D. ... cchi, V. Giovannetti, H. Nakazato, S. Pascazio and K. Yuasa, “Exponential rise of dynamical com... ity in quantum computing through projections”, Nat. Commun. **5**, 5173, 2014.
C. Arenz, D. Burgarth, P. Facchi, V. Giovannetti, H. Nakazato, S. Pascazio and K. Yuasa, “Universal Control Induced by Noise”, PRA **93**, 062308, 2016.

Experimental demonstration at Technion



two transmons

E. Blumenthal, C. Mor, A. A. Diringer, L. S. Martin, D. Burgarth, K. B. Whaley, S. Hacohen-Gourgy, "Demonstration of an entangling gate between non-interacting qubits using the Quantum Zeno effect", arXiv:2108.08549

Experimental demonstration at Technion

$\mathscr{H} = \mathbb{C}_3 \otimes \mathbb{C}_2 \qquad |nm\rangle : n \in \{0,1,2\}, m \in \{0,1\}$

$P = I - |21\rangle\langle 21|$

non-local

continuous measurement: GKLS engineering

$$H = i(|1\rangle\langle 2| - |2\rangle\langle 1|) \otimes I$$

local

$$= i(|1\rangle\langle 2| - |2\rangle\langle 1|) \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|)$$

$$= i|10\rangle\langle 20| + i|11\rangle\langle 21| - i|20\rangle\langle 10| - i|21\rangle\langle 11|$$

$$PHP = i(|1\rangle\langle 2| - |2\rangle\langle 1|) \otimes |0\rangle\langle 0|$$

entangling!

E. Blumenthal, C. Mor, A. A. Diringer, L. S. Martin, D. Burgarth, K. B. Whaley, S. Hacohen-Gourgy, "Demonstration of an entangling gate between non-interacting qubits using the Quantum Zeno effect", arXiv:2108.08549

Gate Quality

continuous measurement formalism:

$$\dot{\rho} = -i[H,\rho] + \gamma \mathscr{D}_P(\rho)$$

$$\mathscr{D}_P(\rho) = P\rho P - (P\rho + \rho P)/2$$

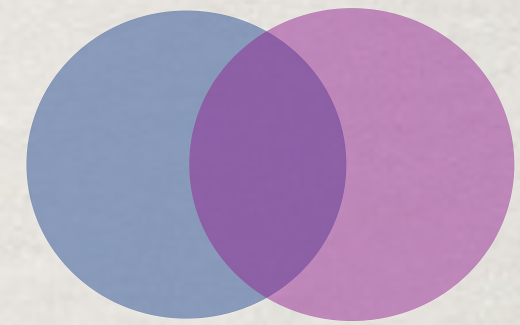
gate error:

$$\epsilon \equiv \| e^{t(\gamma \mathscr{D}_P(\cdot) - i[H,\cdot])} - e^{-it[PHP,\cdot]}(I + 2\mathscr{D}_P(\,\cdot\,)) \|_\diamond$$

$$\Delta h \equiv e_{\max} - e_{\min}$$

$$\epsilon \leq \frac{16\Delta h}{\gamma}\left(1 + t\Delta h + \frac{1 - e^{-t\gamma/2}}{2}\right) + e^{-t\gamma/2}$$

Experiment: ~70%



Zeno+GKLS works!

E. Blumenthal, C. Mor, A. A. Diringer, L. S. Martin, D. Burgarth, K. B. Whaley, S. Hacohen-Gourgy, "Demonstration of an entangling gate between non-interacting qubits using the Quantum Zeno effect", arXiv:2108.08549

But there is some friction…

$$e^{(\gamma\mathcal{L}_0+\mathcal{L})t} = e^{\gamma\mathcal{L}_0 t}e^{\mathcal{L}_Z t}\mathcal{P}_\varphi + \mathcal{O}(1/\gamma)$$

$$\mathcal{L}_Z = \sum_{\alpha_k\in i\mathbb{R}} \mathcal{P}_k\mathcal{L}\mathcal{P}_k, \quad \mathcal{L}_0 = \sum_k(\alpha_k\mathcal{P}_k + \mathcal{N}_k), \quad \mathcal{P}_\varphi = \sum_{\alpha_k\in i\mathbb{R}} \mathcal{P}_k$$

- Obviously $\Lambda(t) \equiv e^{\mathscr{L}_z t}\mathscr{P}_\varphi$ is CPTP
- $\Lambda(t)\Lambda(s) = \Lambda(t+s)$ but $\Lambda(0) = \mathscr{P}_\varphi$
- Curiously, $e^{\mathscr{L}_z t}$ is not always CPTP and $\mathscr{L}_z$ not GKLS
- By Christensen/Evans, it can be modified to $\mathscr{G}_z$ of GKLS form such that $e^{\mathscr{G}_z t}\mathscr{P}_\varphi = e^{\mathscr{L}_z t}\mathscr{P}_\varphi$
- I don't know the general strategy to do this. Dariusz?

Conclusions

- Quantum Zeno Dynamics can be used to create new effective generators through a variety of mechanisms:

  - measurements
  - strong dissipation
  - strong fields
  - unitary kicks
  - CPTP kicks

  Adiabatic Theorem

- These have applications in Quantum Control and Quantum Computation
- Experiments catching up
- We connected two results by Sudarshan, but not without friction