Classifying quantum dynamical maps: an approach from Past-Future Correlations

Adrián A. Budini
Centro Atómico Bariloche
CONICET, Argentina

Workshop: Celebrating the 60th anniversary of Sudarshan's paper on dynamical maps

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Stochastic Dynamics of Quantum-Mechanical Systems

E. C. G. Sudarshan*
Department of Physics and Astronomy, University of Rochester, New York

P. M. Mathews
Department of Physics, University of Madras, Madras, India

and

Jayaseetha Rau†
Department of Physics, Brandeis University, Waltham, Massachusetts

(Received August 15, 1960)

The most general dynamical law for a quantum mechanical system with a finite number of levels is formulated. A fundamental role is played by the so-called "dynamical matrix" whose properties are stated in a sequence of theorems. A necessary and sufficient criterion for distinguishing dynamical matrices corresponding to a Hamiltonian time-dependence is formulated. The non-Hamiltonian case is discussed in detail and the application to paramagnetic relaxation is outlined.

$$\rho_0 \to \rho_t = \mathcal{A}[\rho_0] \qquad (1)$$

- **Central ingredients of the theory of open quantum systems are there!**
- **Conditions on the "dynamical matrix" A that guaranty physical evolutions; Unitary evolutions; (statistical) mixture of dynamics; Relaxation :**
- **Classification of quantum dynamical maps**

Classification of quantum dynamical maps

- A completely positive condition guarantees mapping a density matrix into a density matrix (start to be defined in Sudarshan et. al. paper)
- **Quantum Markovian maps** vs. **Quantum Non-Markovian maps (?)**
- *If non-Markovian*, is there any **environment-to-system backflow of information?**
- The previous questions has been a research topic in the last ten years.
- Non-operational approaches: only rely on the dynamical map properties
- Operational approaches: in addition, the system is measured at different times
- **Goal of the talk: to answer the previous questions about maps classification with an operational based approach.**

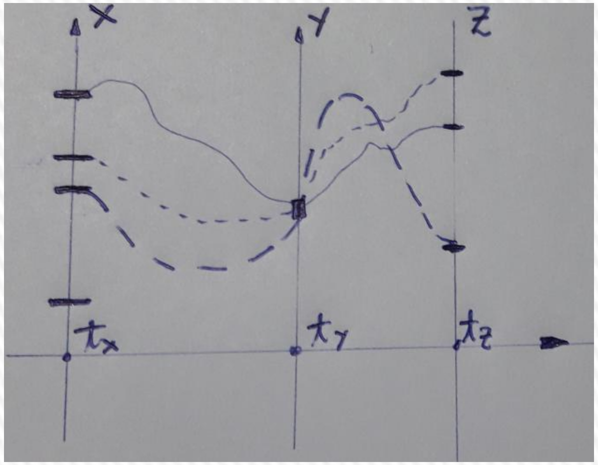
Markovianity and non-Markovianity

- **Markovianity (conditional probabilities)**

$$P(X_n|X_{n-1},\cdots X_2,X_1) = P(X_n|X_{n-1}) \quad (2)$$

- **How this property can be tested?**
- **Which is the statistical relation between past events (X) and future ones (Z) when conditioned to a fixed intermediate state (Y)?**

MARKOV: CONDITIONAL PAST-FUTURE STATISTICAL INDEPENDENCE



past **present** **future**

Via Bayes rule:

$$P(z,x|y) = P(z|y)P(x|y) \quad (3)$$

Markovian

$$P(z,x|y) = P(z|y,x)P(x|y) \quad (4)$$

Non–Markovian

memory effects

can be detected with three measurements

Environment-to-system back flow of information

- **The definition must be valid in classical and quantum domains**

- **Information stored in the environment influence the system at later times**

- **Given system (s) and environment (e), with density matrix $\rho_{se}(t)$**

(5) $$\rho_s(t) = \mathrm{Tr}_e[\rho_{se}(t)]$$

$$\rho_e(t) = \mathrm{Tr}_s[\rho_{se}(t)]$$ **(6)**

- **<u>Definition</u>: If $\rho_e(t)\ [and\ \frac{d\rho_e(t)}{dt}]$ does not depend on the system degrees of freedom, there is not a "physical" environment-to-system backflow of information.**

<u>Bidirectional information flows</u> imply that the environment density matrix must depend on the system degrees of freedom.

(*)For classical noisy environments there is not any bidirectional flow

System Projective measurements

➢**Which experimental procedure allows to detect the previous properties?**

➢**A projective system measurement (projectors $|y\rangle\langle y|$) implies**

$$\rho_{se} \to |y\rangle\langle y| \otimes \frac{\langle y|\rho_{se}|y\rangle}{\mathrm{Tr}_{se}[|y\rangle\langle y|\rho_{se}]} \quad (7)$$

➢If the <u>post-measurement bath state</u> depends on the previous system history: one must to detect memory non-Markovian effects.

(8)

➢After the measurement, introduce the <u>random system change</u> $|y\rangle\langle y| \to |\breve{y}\rangle\langle\breve{y}|$ and <u>disregard the outcomes y</u>:

$$\rho_{se} \to |\breve{y}\rangle\langle\breve{y}| \otimes \sum_y \langle y|\rho_{se}|y\rangle = |\breve{y}\rangle\langle\breve{y}| \otimes \mathrm{Tr}_s[\rho_{se}]. \quad (9)$$

If the environment does not depend on the system, memory on the system history has been deleted: <u>(none back flow information) => (memoryless)</u>

➢These properties allows to establishing an experimental procedure for detecting the previous definitions of memory and information flows.

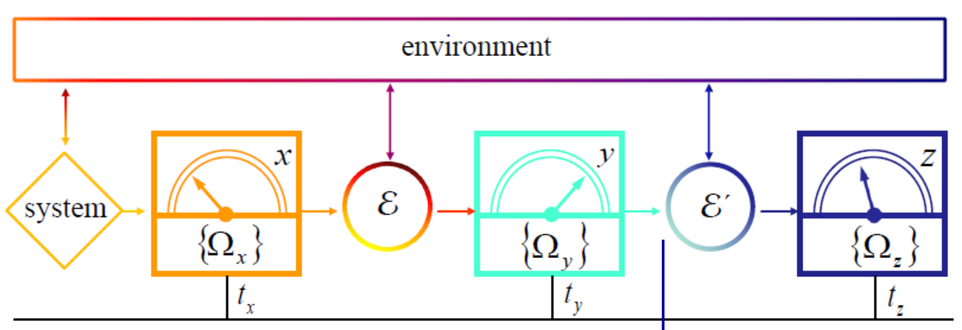
Conditional Past-Future (CPF) Correlation

Measurment operators

$\{\Omega_x\}, \{\Omega_y\}, \{\Omega_z\}$

Dynamical maps $\mathcal{E}$

(the same as in Sudarshan et. al. paper)



$t = t_y - t_x, \quad \tau = t_z - t_y,$

$$C_{pf}(t,\tau)|_y = \sum_{z,x} (z\ x)[P(z,x|y) - P(z|y)P(x|y)] \quad (10)$$

$C_{pf} = 0 \Leftarrow$ Markovian

$C_{pf} \neq 0 \Rightarrow$ Non–Markovian

Deterministic scheme (11)

$C_{pf} = 0 \Leftarrow$ Absence of Bidirectional information flows

$C_{pf} \neq 0 \Rightarrow$ Presence of Bidirectional information flows

$|y\rangle\langle y| \rightarrow |\breve{y}\rangle\langle\breve{y}|$

Random scheme (12)

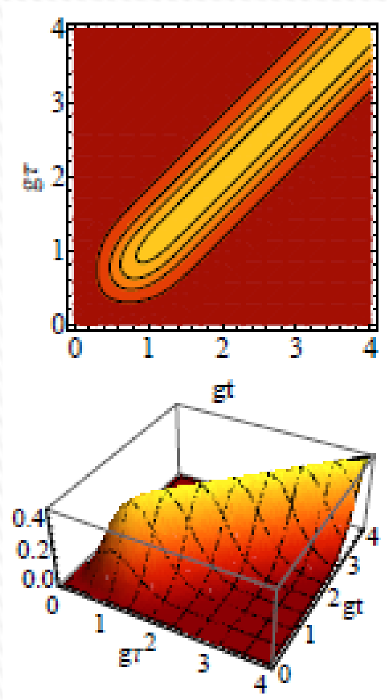
Memory effects in the dephasing of a qubit

$$H_T = \sigma_{\hat{z}} \otimes \sum_{k=1}^{N} g_k \sigma_{\hat{z}}^{(k)} \quad (13)$$

$$C_{pf}(t,\tau) = f(t,\tau) - f(t)f(\tau)$$

$f(t)$ **Coherence decay**

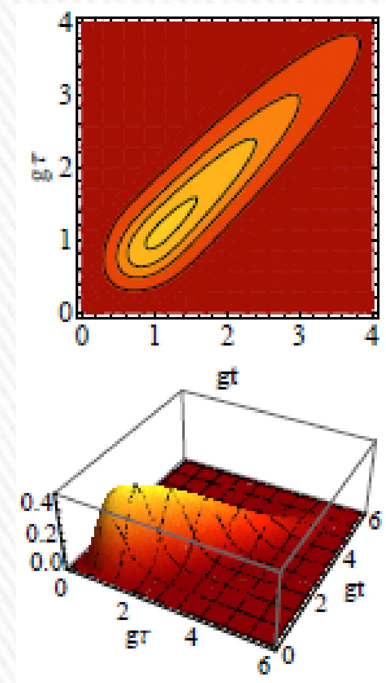
$$f(t,\tau) = f(t+\tau) + f(t-\tau) \quad (14)$$



$$\lim_{t\to\infty} C_{pf}(t,t) \neq 0$$

Infinite bath-correlation-time

Spin bath Model



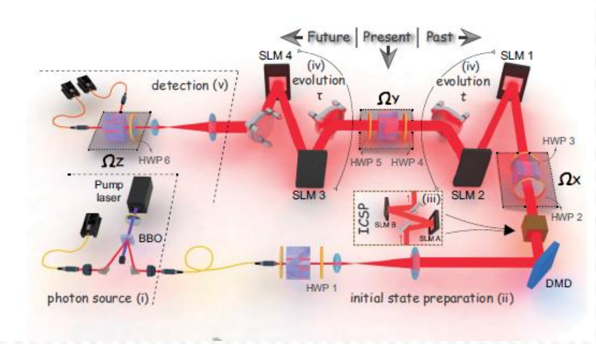
$$\lim_{t\to\infty} C_{pf}(t,t) = 0$$

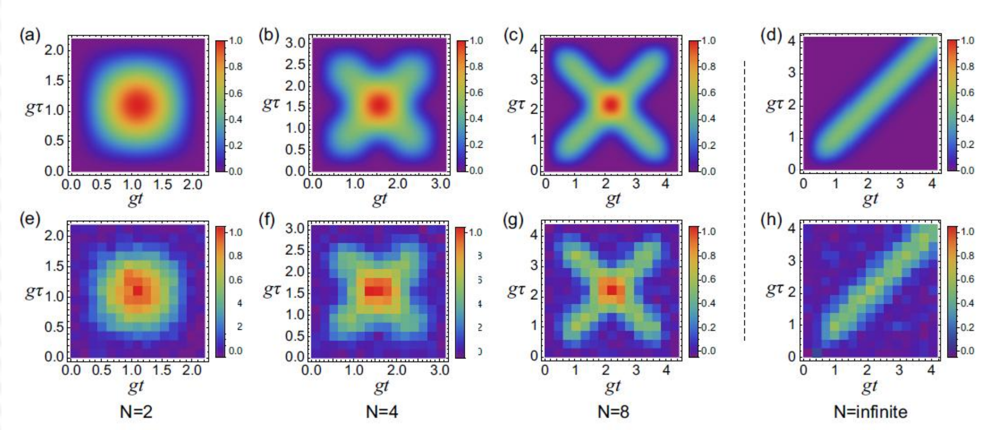
Finite bath-correlation-time

Hamiltonian Noise model

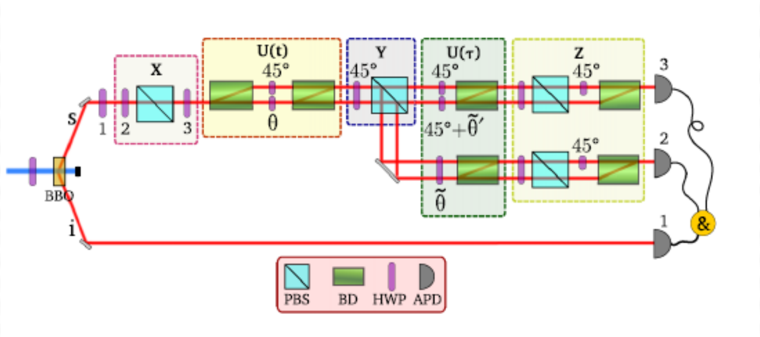
PRL (2018)

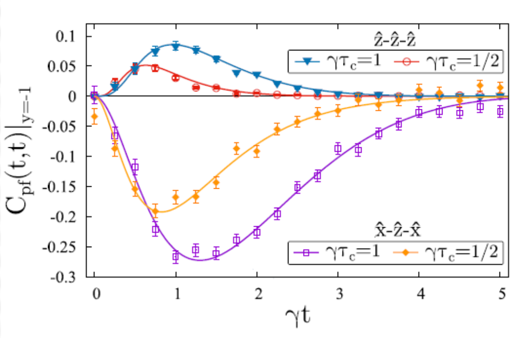
Experimental quantum-optical implementations





Hefei (China) quantum optics group_PRA 100 (2019): **The CPF allows to detect initial system –environment correlations**





Rio de Janeiro (Brazil) quantum optics group_PRA 101 (2020):
Detection of memory effects close to the validity of Born-Markov approximation (two-level system with a bosonic bath Temp=0)

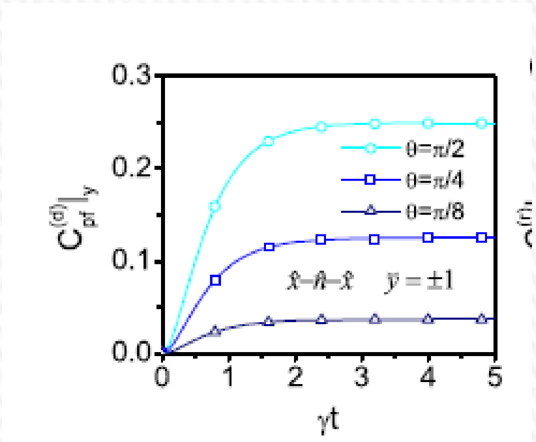
Absence of bidirectional system-information flows

(15) $$\frac{d\rho_t}{dt} = \frac{1}{2}\sum_{\alpha=\hat{x},\hat{y},\hat{z}} \gamma_\alpha(t)(\sigma_\alpha\rho_t\sigma_\alpha - \rho_t),$$

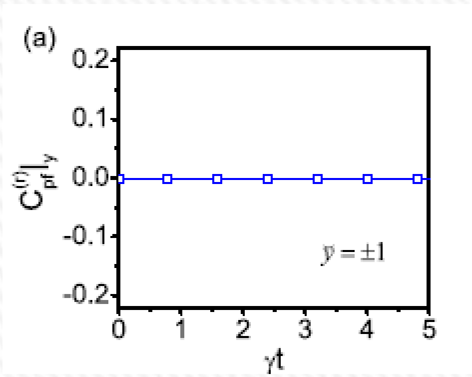
Eternal non-Markovian evolution: one rates is always negative

Relies on a random superposition of Markovian dynamics

(same as in Sudarshan et. al. paper)



CPF detects memory effects



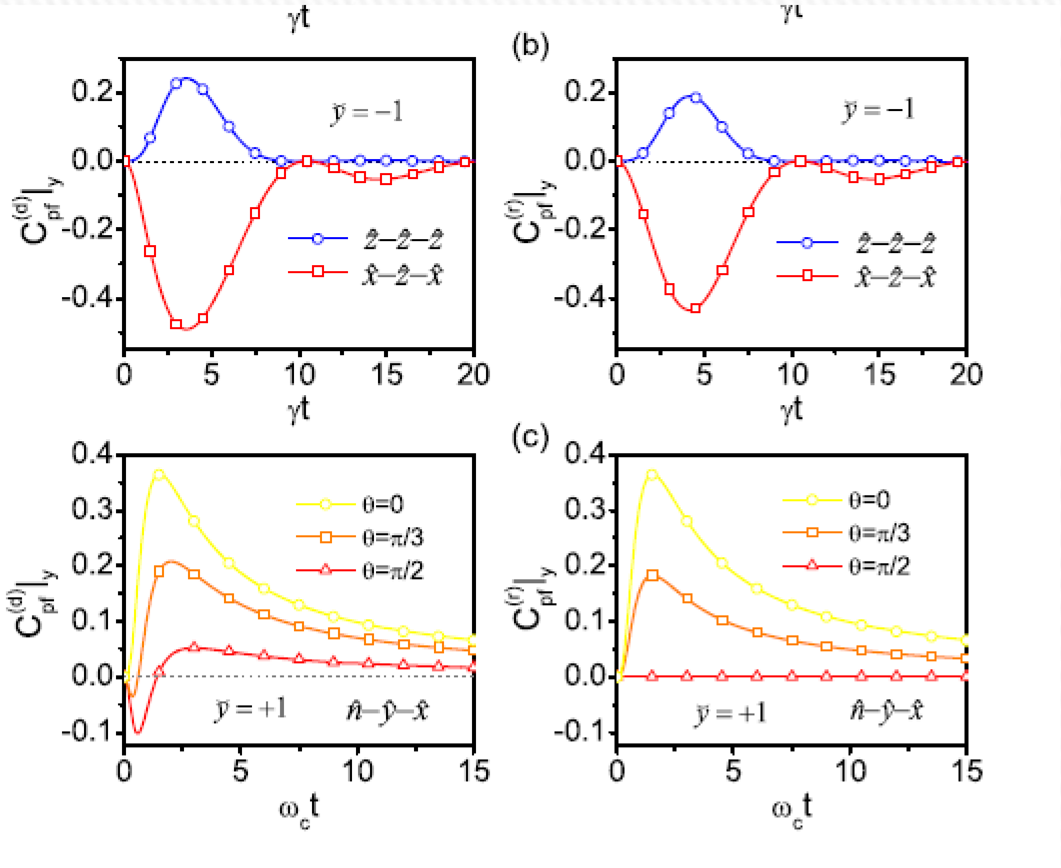
After randomizing the intermediate post-measurement state, CPF=0

There is not bidirectional information flows

PRA (2021)

Presence of bidirectional system-information flows

Standard CPF | **CPF with random post -measurement state**



Dephasing coupling

Dissipative coupling

The system is a two-level model coupled to a bosonic bath at zero temperature

Conclusions

- A scheme for classifying dynamical map was established:
  *Markovian Vs non-Markovian
  *System-environment bidirectional information flows
- The proposal relies on a quantum extension of a conditional past-future independence valid for classical Markovian process
- The conditional past-future correlation (CPF) relies on a minimal set of three quantum measurements (and post-selection)
- Experimental implementations has been performed in quantum optics
- An experimental procedure for detecting of bidirectional information flows is established
- Analysis on different possible "dynamical maps" under work