South African Mathematical Sciences Landscape in 2050

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Legacy of Apartheid on our Education

"We should not give the Native any academic education. If we do, who is going to do the manual labour in the community"

[**JN le Roux in 1945**]

"There is no place for [the Bantu] in the European community above the level of certain forms of labour….What is the use of teaching the Bantu child mathematics when it cannot use it in practice. This is quite absurd. Education must train people in accordance with their opportunities in life according to the sphere in which they live".

[**Hendrik Verwoerd in 1953**]

"*I have no consulted the African people on the language issue and I'm not going to. An African might find that 'the big boss' only spoke Afrikaans or only spoke English. It would be to his advantage to know both languages"*

[**Janson Punt in 1974**]

Presentation Outline

- Introduction
- Mathematical Sciences in 2025
- SA Mathematical Sciences Research Landscape: 2011 – 2020
- Mathematical sciences knowledgebase
- People pipeline
- Knowledge exchange
- Concluding remarks and shopping list

INTRODUCTION

- The presentation was originally inspired by a report released in 2013 titled "*The Mathematical Sciences in 2025"* commissioned by the US National Research Council of the National Academies of Science, Engineering and Medicine;
- Publication coincided with the end of my term as Vice-Chancellor of Wits University and on a "*retooling leave";*
- Fellowship from Stellenbosch Institute for Advanced Studies: October 2013 and March 2014;
- **Project:** How had South African Mathematical Sciences Research evolved since 2000;
- I was invited to write a short article by the Editor of the Notices of the American Mathematical Society on "*the past, present and future of South African Mathematics*" (Eder Kikianty and Loyiso Nongxa: February 2022 Vol 69, No 2. Black History Month)

INTRODUCTION

- **Referee's remark**: the section on "the future of SA Mathematics" belongs to a ***National Strategy for the Mathematical Sciences in South Africa.***
- We do not have one and never had one;
- Who is responsible for a **National Strategy for the Mathematical Sciences**?
- Who is responsible for the health and vibrancy of academic disciplines?
- Who is responsible for benchmarking SA's Mathematics against global trends?
- Would it the National Research Foundation or the Department of Science and Innovation or the Academy of Sciences of South Africa or the South African Mathematical Society or the South African Statistical Association or the South African Mathematics Foundation or the Association for Mathematics Education of South Africa
- Maybe all or some or none of the above?

The Mathematical Sciences in 2025

- *"Much of twenty-first century science and engineering is going to be built on a mathematical sciences foundation";*
- *"The mathematical sciences provide the fundamental language for computational simulation and data analysis";*
- *"The mathematical sciences are increasingly fundamental to the social sciences and have become integral to many emerging industries"*; (cf: "***Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy***" by Cathy O'Neill; or Mathematics of Social Networks).
- *"Support the health of the mathematical sciences ecosystem and a robust educational pipeline"*;
- *"Increase public awareness of the expanding role of the mathematical sciences"*;

Attributes: Mathematicians in 2025

- “Knowledgeable across a broad range of disciplines, beyond their own area(s) of expertise”;
- “Communicate well with researchers in other disciplines”;
- “Understand the role of the mathematical sciences in the wider world of science, engineering, medicine, defence, and business”;
- “Have some experience with computation”;

Key drivers: Mathematical Sciences in 2025

- Computing;
- Big Data;
- Increasing complexity;
- Uncertainty and risk;
- Connectedness.

Future developments: Mathematical Sciences in 2025

- Simulation;
- Information Science;
- High dimensionality and large data sets;
- Mathematics of physical systems;
- Mathematical modelling;
- Computational and statistical reasoning;
- Imaging.

(Has this influenced the National Science Foundation funding of Mathematical Sciences Division?)

Mathematical Sciences research landscape: 2011 - 2020

- Extracted (**by hand**!) all articles in MathSciNet for the years 2011 to 2020 with at least one SA author;
- Clustered the MSN categories into 15 categories aligned to 2010, 2014 and 2018 ICM sessions: Logic and Foundations; Algebra; Geometry; Number Theory; Topology and Category Theory; Analysis; Functional Analysis and Operator Theory; Differential Equations; Mathematical Physics; Statistics, Probability and Stochastic Processes; Combinatorics; Theoretical Computer Science; Numerical Analysis and Scientific Computing; Control Theory and Optimisation; Mathematics in Science and Technology.
- Assigned each rated researchers to at least one of 15 categories using their articles published between 2011 and 2020.
- Number of articles in MathSciNet for the years 2011 – 2020 with at least one author in each category.

Observations: research areas

- **Foundations and Logic:** Just over 60% of all articles published were devoted to problems within three research areas, namely general and algebraic logic as well as (distributive) lattices, in the form of frames and locales.
- **Algebra:** A large number of the rated people is concentrated mainly around three sub-themes: near-rings; radicals and radical properties of associative rings; and representation theory of finite groups.
- **Geometry:** This cluster had the lowest number of publications during the period under consideration and almost 60% of publications were in differential geometry and some articles were authored by researchers working in, for example, mathematical physics. All the other major topics which are included in this cluster (for example algebraic geometry, convex and discrete geometry) are either under-represented or totally non-existent.

Observations: research areas

- **Number Theory**: The main topics covered in the published articles are elementary number theory; sequences and sets; diophantine equations and multiplicative number theory.
- **Topology and Category Theory:** In topology, the research topics mainly fall under general topology, asymmetric topology and point-free topology. Unlike previous years, there were no rated mathematicians in algebraic topology and research in all aspects of algebraic topology almost non-existent, only 5 publications during the ten-year period. Research in category theory covers a broad spectrum of topics that fall under this cluster in the MSC2020 Category Section.
- **Analysis:** Almost 80% of the publications fall under three areas: real functions (functions of one variable and inequalities in real analysis), special functions (hypergeometric functions) and dynamical systems and ergodic theory.

Observations: research areas

- **Functional Analysis and Operator Theory:** Just over 70% of publications are in operator theory with at least 50% in nonlinear operators. Other topics include normed linear spaces and Banach spaces, topological algebras and self-adjoint operator algebras.
- **Differential equations:** The research pursued was mainly in general and qualitative properties of differential equations, in particular symmetries and invariants of ordinary and partial differential equations; as well as partial differential equations of mathematical physics and other areas of application.

Observations: research areas

- **Mathematical Physics:** This cluster contains a number of areas that, on their own, are considerably stronger than a few of the clusters in our list. These are fluid mechanics, quantum theory; and relativity and gravitational theory. Research in quantum theory was mainly concentrated on foundations, quantum field theory and other related classical field theories; and mathematical methods in quantum theory. In relativity and gravitational, research was mainly focused on general relativity and relativistic cosmology. In fluid mechanics, the most visible topics included foundations, incompressible viscous fluids, flows in porous media; and magnetohydrodynamics and electro-hydrodynamics.

Observations: research areas

- **Statistics, Probability and Stochastic Processes: T**his cluster had the largest number of rated researchers including seven who enjoyed considerable international recognition. Although about a third are at or past retirement age, there is also a number of younger researchers active within this cluster. The research covers a broad spectrum of specialties, although there are some topics not covered. Most of the research activity was focused mainly in parametric and nonparametric inference, multivariate analysis and applications of statistics, for example, to actuarial sciences and finance, to biology and medical sciences and applications to social sciences.

Observations: research areas

- **Combinatorics**: About 94% of publications during the period were in two areas: **enumerative combinatorics** and **graph theory**. The share of graph theory publications within the combinatorics cluster almost 80%. What is striking is that at least 40% of graph theory publications are within one subspecialty, namely vertex subsets with special properties (dominating sets, independent sets, cliques, etc.). Consequently many topics are either underrepresented or do not appear at all. These would include geometric and topological aspect of graph theory; graphs and linear algebra (matrices, eigenvalues, etc.); graph operations (line graphs, products, etc.); random graphs (graph-theoretic aspects); small world graphs, complex networks (graph-theoretic aspects) and graph algorithms (graph-theoretic aspects). Research in enumerative combinatorics occurred in combinatorial aspects of partitions of integers, exact enumeration problems and to a lesser extent in combinatorial probability.

Observations: research areas

- **Numerical Analysis and Scientific Computing:** Numerical Analysis has been an area of steady, organised and continued research activity for many years, preceding the introduction of the rating system. The number of rated researchers is relatively low, compared to the other areas of research strength with only one researcher who enjoyed international recognition. Slightly over 70% of all the publications were in numerical methods for ordinary and partial differential equations, the other noteworthy research area being nonlinear algebraic or transcendental equations.
- **Mathematical Aspects of Computer Science:** This area has a fairly low number of rated researchers and two were regarded as world leaders. There was also a low number of publications compared to the other clusters; the work of the A-rated researchers was mainly in combinatorics. The most prominent areas of research were theory of computing and discrete mathematics in relation to computer science.

Observations: research areas

- **Mathematics in Science and Technology:** During the period under consideration, there was an explosion of research output in mathematical biology. It has one of the largest concentration of rated researchers, five of whom enjoy considerable international recognition and one was regarded as one of the world leaders in his field. In terms of the Mathematics Subject Classification, more than 90% of the publications were in physiological, cellular and medical topics; and genetics and population dynamics.

Benchmarking against global trends

- How do we benchmark South African mathematical sciences research? Through periodic independent reviews?(only once before) Scientometrics (2 reports since 2018)? NRF ratings?

Personal preferences:

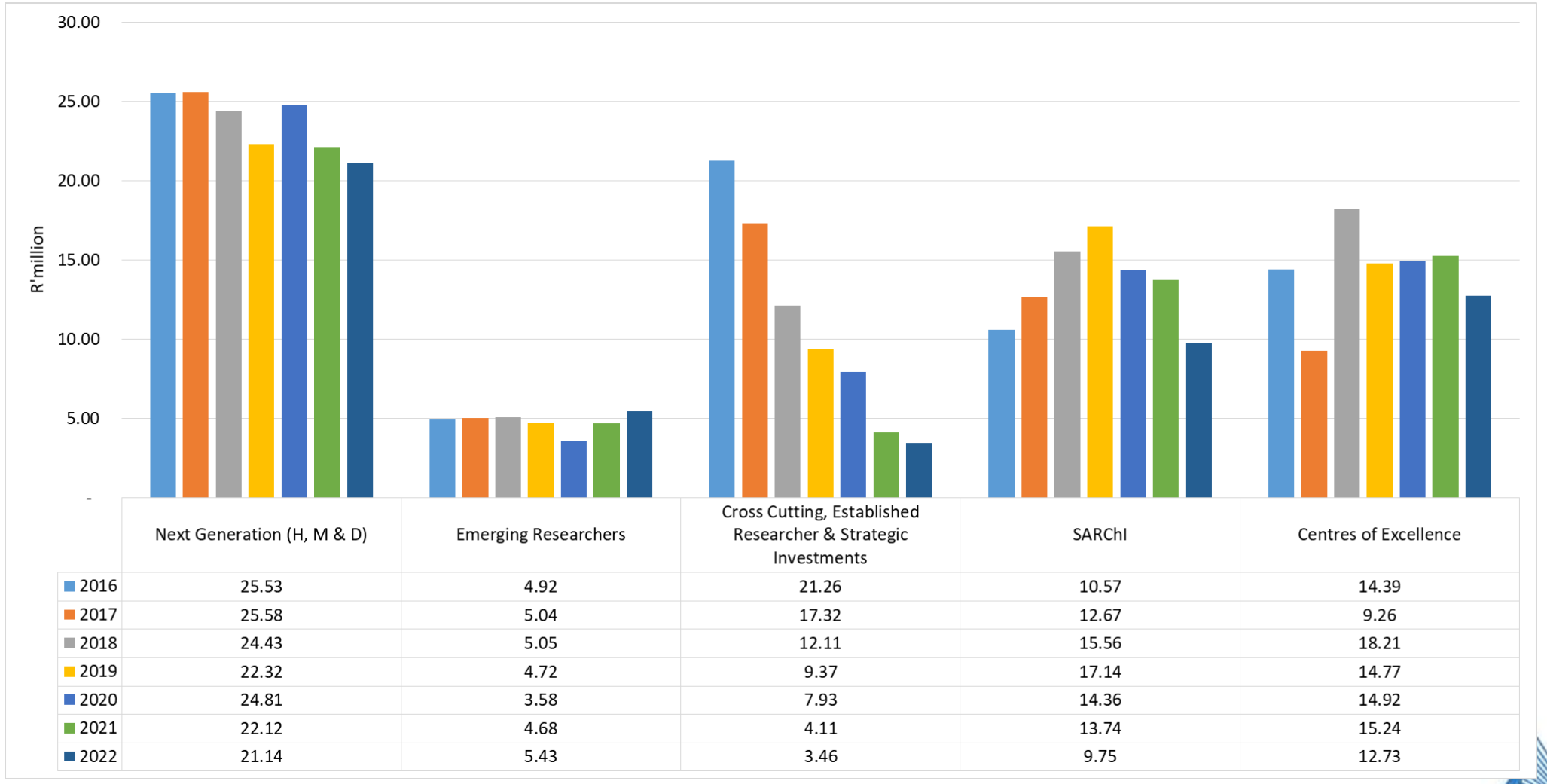
- Topics of invited lectures (plenary and sessional) at the International Congresses of Mathematicians;
- Semester programmes hosted by reputable research institutes (Global North hegemony?);
- Themes funded by Global Research Funders (Global North hegemony?)
- Attractiveness of South Africa to continental (and global) mathematical sciences talent;
- Level of connectedness between producers of mathematical intellectual capital (academia) and consumers of that capital.

Transforming the South African mathematical sciences knowledgebase

We can hopefully shape the future research landscape through:

- interventions around current early career mathematical scientists;
- vacancies that will arise through retirements;
- transforming graduate education in the mathematical sciences
- Who will be teaching?
- What will be taught?
- Those at most 40 years old in the system will be the leaders of the system by 2050;
- Unless we take deliberate steps to transform the current knowledgebase, then **most probably** by 2050 the landscape will resemble what we currently have;

NRF Investment in Mathematics and Statistics: Researcher Pipeline and Strategic Interventions



	Next Generation (H, M & D)	Emerging Researchers	Cross Cutting, Established Researcher & Strategic Investments	SARChI	Centres of Excellence
2016	25.53	4.92	21.26	10.57	14.39
2017	25.58	5.04	17.32	12.67	9.26
2018	24.43	5.05	12.11	15.56	18.21
2019	22.32	4.72	9.37	17.14	14.77
2020	24.81	3.58	7.93	14.36	14.92
2021	22.12	4.68	4.11	13.74	15.24
2022	21.14	5.43	3.46	9.75	12.73

Source:

National Research Foundation's Research and Development Information Platform (RDIP).

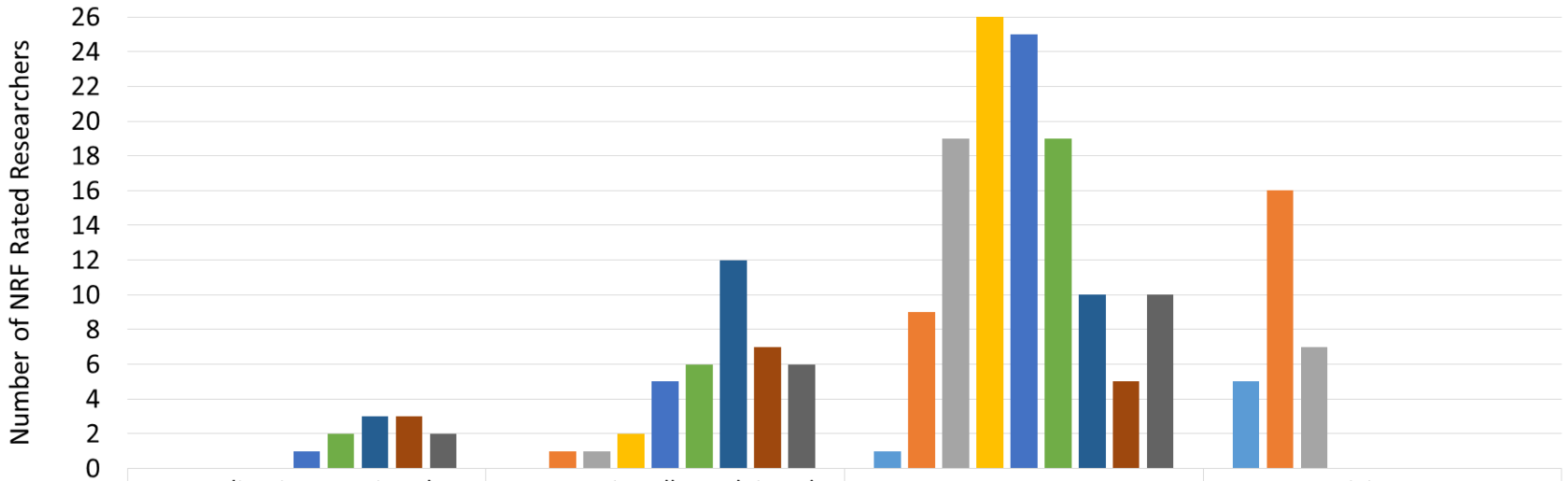
Data from 2023 CREST Reports

- In 2020 there were 528 mathematics academics: (approximately) 61% with a doctoral qualification; 32% female; 38% White; 62% more than 50 years old. More than 300 vacancies by 2040.
- In 2020 there were 255 statistics academics: (approximately) 45% with a doctoral qualification; 40% female; 47% White; (only) 27% are more than 50 years old.
- **Mathematics:** will individual departments look at consolidating (building on) their research specialisations in filling the arising vacancies? If so, then research landscape remains the same.
- **Or:** can they act 'collectively' in diversifying existing research concentrations and also creating new areas of research specialization?
- **Statistics:** since at least 70% are below the age of 50, it's likely that by 2050 research specialisations will reflect current landscape.

Data from NGA(MaSS) database

- Around 330 have been or are potential beneficiaries; academics without PhDs and those within 5 years of completing their doctoral degrees, irrespective of age.
- 54.3% in Mathematics; 44.2% in Statistics and 3.5% in Data Science, Machine Learning or Computer Science.
- 109 were/are registered for PhDs (34 since establishment of NGA);
- 160 PhD holders (135 completed since 2017);
- 33 were/are registered for Masters (24 obtained their honours qualification between 2009 and 2016);
- 31 not registered (18 hold Masters and 13 honours degree).

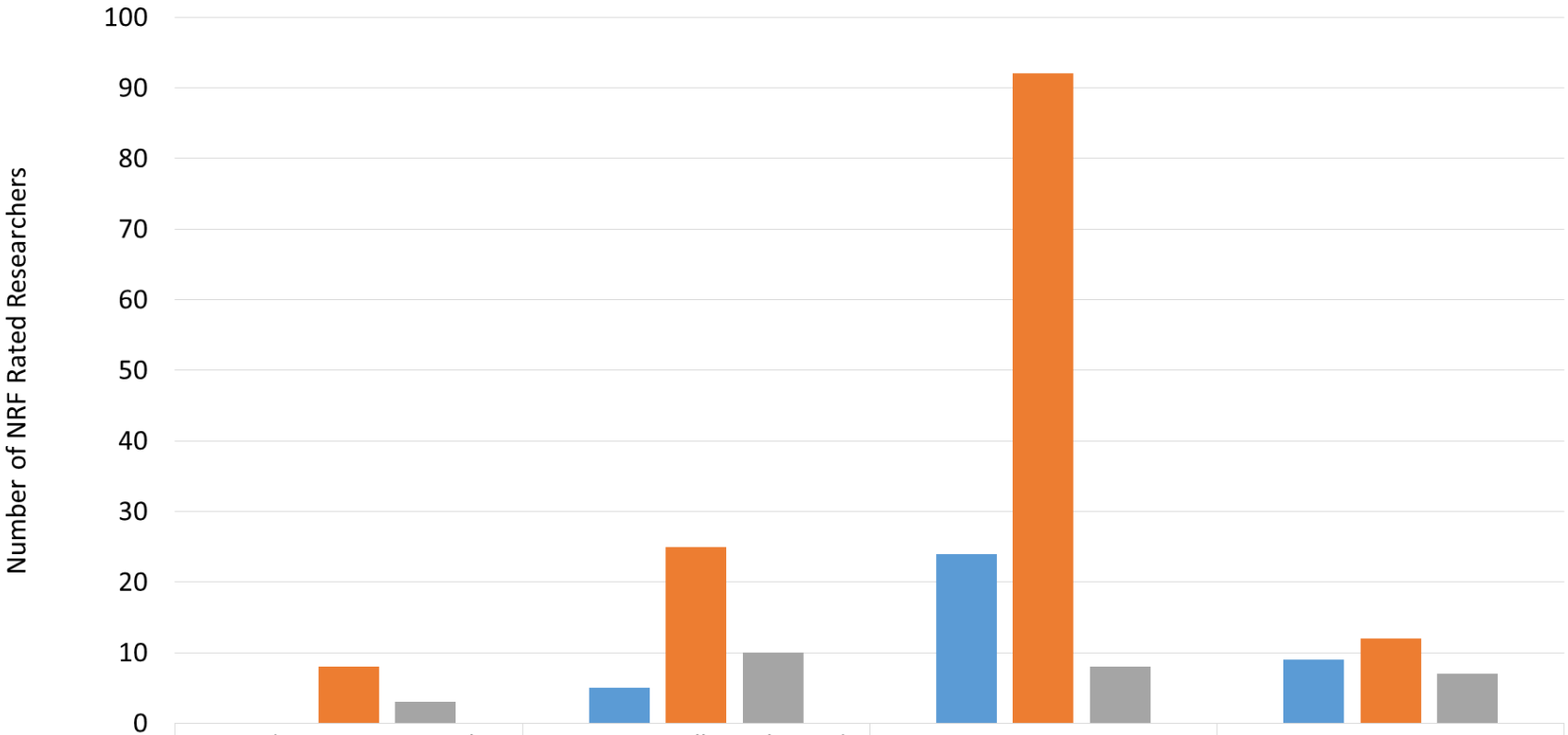
Number of NRF Rated Researchers in Mathematics & Statistics (2022) – Age Profile



	Leading international researchers	Internationally acclaimed researchers	Established researchers	Promising young researchers
30 - 34			1	5
35 - 39		1	9	16
40 - 44		1	19	7
45 - 49		2	26	
50 - 54	1	5	25	
55 - 59	2	6	19	
60 - 64	3	12	10	
65 - 69	3	7	5	
70 +	2	6	10	

Source: *National Research Foundation's Research and Development Information Platform (RDIP).*
** Based on number of researchers allocated to Mathematics related rating panels.*

Number of NRF Rated Researchers in Mathematics & Statistics (2022) – Gender Profile



	Leading international researchers	Internationally acclaimed researchers	Established researchers	Promising young researchers
SA Women		5	24	9
SA Men	8	25	92	12
Foreign	3	10	8	7

Source: *National Research Foundation's Research and Development Information Platform (RDIP).*
** Based on number of researchers allocated to Mathematics related rating panels.*

Mathematical Sciences Graduate Education

- Non-existent.
- Choice of advanced topics constrained by institutional specialization; little to no opportunity for broader exposure to important topics;
- Students specialise TOO EARLY. A topic for an honours project often becomes a lifetime specialty;
- Areas of contemporary research activity in South Africa (therefore producing Masters and PhDs) evolved in the1970s and 1980s and have remained static since the 1990s;
- Very few of SA's areas of research concentration feature (prominently) in the ICM plenary and sessional invited lectures.
- Without deliberate interventions contemporary landscape will be replicated by middle of this century.

A case for Graduate Coursework

- Informal partnership between NITheCS's South African Theory and Computation School (SATACS) and NGA-Coursework.
- A national platform for offering introductory and advanced topics in the mathematical sciences;
- PhD-preparatory programmes; "Hot Topics" programme; one instrument to transform the current mathematical sciences knowledgebase and the national research landscape
- NGA(MaSS) Knowledgeshare grant from Oppenheimer Memorial Trust used to fund course offerings;
- **Heavy lifting: a** plethora of issues that need to be addressed: coursework at Masters and PhD level; credit transfer; different credits for 'similar topics'; workload models; teaching subsidy; etc.

Mathematical Sciences for Innovation and Development

From the February 2016 “Basic Sciences Development and Support Framework:

- “*For global competitiveness and the generation of employment opportunities, our country needs to develop long-term sustainable strategies for support of Basic Sciences (BS)”*
- *the Basic Sciences refer to the scientific disciplines where fundamental knowledge about the natural and the physical world is built and maintained, and covers chemistry, physics, mathematics and statistics as well as computer science, biological and geological sciences…”*
- *“…the support for the BS in South Africa (SA) is currently unstructured and in some instances insufficient”; as a result the related disciplines and the associated science, engineering and technology (SET) fields they underpin are negatively affected”.*

Mathematical Sciences for Innovation and Development

In the 21st century, the new sectors of vibrant and dynamic activity that are engines for economic growth globally are founded on high level mathematical and statistical sciences. Mathematics and Statistics are sources of ideas for advanced algorithms that drive Google, Facebook, Amazon and similar such companies. Mathematics and Statistics are crucial in machine learning, robotics and artificial intelligence. Optimisation is ubiquitous in countless industries: design engineering and manufacturing, logistics, retail, finance, transport and it is a source of productivity and efficiency gains. Fraud has reached epidemic proportions in South Africa and has cost our country billions of rands. Graph theory, a subdiscipline of mathematics going back centuries, and network science are used in fraud detection in private and public sectors. Ideas from mathematics and statistics are key in data compression, data transmission and data security. The foundations of Big Data Analysis are in mathematics, statistics and computer science. More effective business decisions are made by combining experience with statistical analysis. Statistical and Mathematical Modelling and simulation significantly reduce cost in manufacturing industry.

CONCLUDING SHOPPING LIST

- **Reimagine** South African Graduate education (post bachelor's degrees): commission a report or study to examine global trends;
- Create an environment nationally to foster collaboration and deploy existing expertise to serve the whole system;
- Support postgraduate training networks involving local and global partners to seed emerging and contemporary areas of the mathematical sciences;
- **Can NITheCS make an investment in mathematical sciences underpinning its research agenda, for example, Theoretical Foundations of ML and DS?**
- The pool of full-time academics who are not PhD-holders provides an opportunity to transform the national knowledgebase in the mathematical sciences;
- Incentivise those not keen to pursue further degrees to move to TVET colleges or high schools where there's a shortage of highly qualified mathematical sciences teachers and lecturers;
- **Convene a national meeting of early career and senior academics; DHET and DSI officials; current and potential funders of the mathematical sciences; existing national entities to debate how might a 2050 mathematical sciences landscape look like.**

AND LASTLY……

CONSTRUCTIVE CRITIQUE MOST WELCOME.

Thank You

National Graduate Academy:
Mathematical and Statistical Sciences