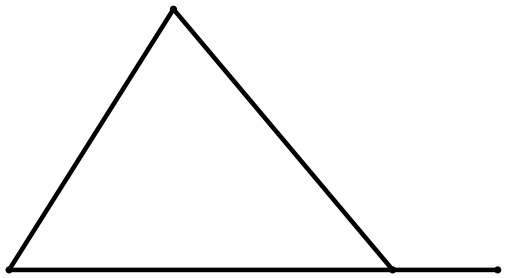
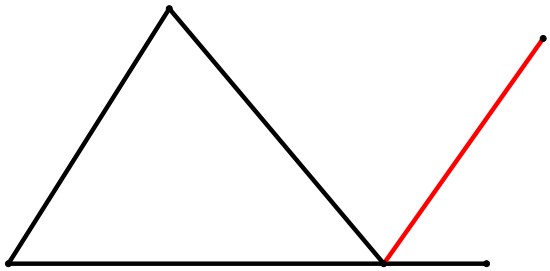
A BRIEF TOUR OF GEOMETRY, GRADES 9,10,11

Poobhalan Pillay, 14 October 2023

(accompanied with recoded audio)





Congruency

Two Triangles are congruent if they are identical in every respect.

There are six numbers associated with every triangle, the three sides and the three angles.

If three numbers of one triangle are the same as the corresponding three numbers of a second triangle , then all six numbers are same

Then three numbers are called conditions for congruency. There are four conditions:

(s, s, s)

(s, $\angle$, s)

($\perp$,h,s)

(($\angle$,s,$\angle$)

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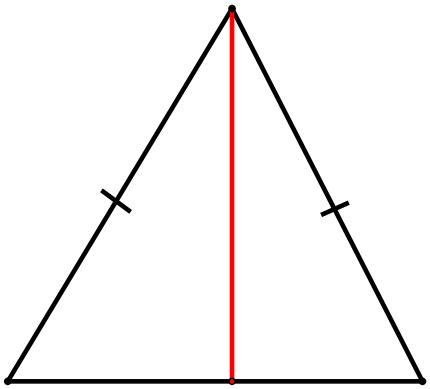
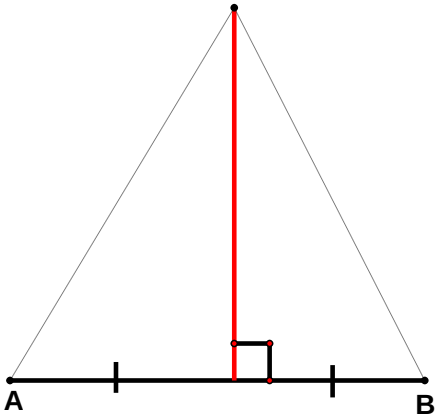
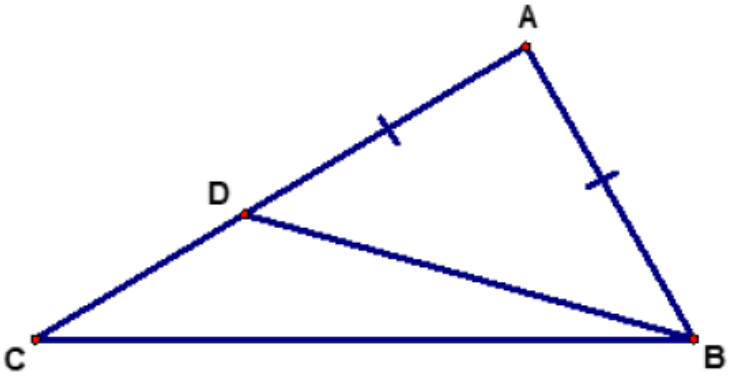
Isosceles Triangles



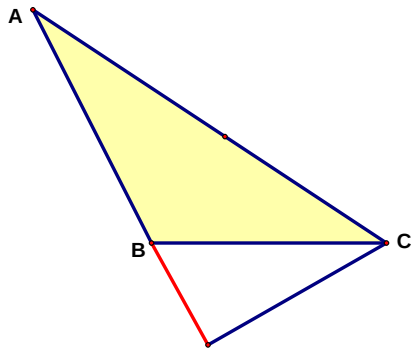


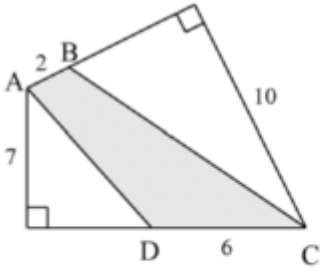
In ΔABC point D is on AC and AD = AB. It is also known that $\angle ABC - \angle ACB = 30°$. Calculate $\angle CBD$.



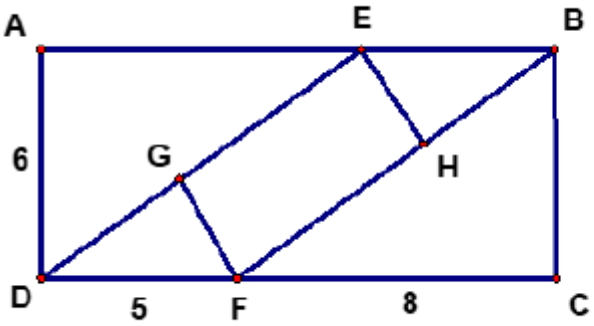
Area of a triangle one half base times height

Area of a parallelogram is equal to base times height



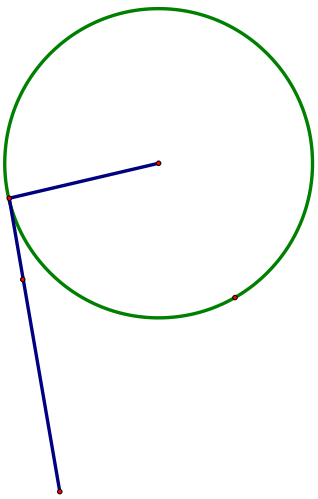


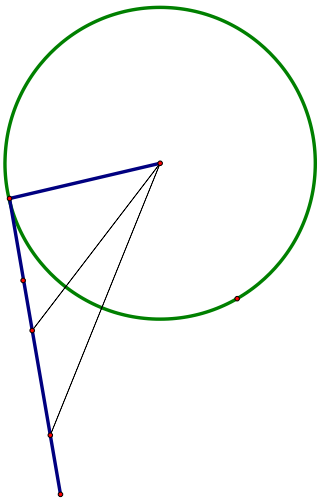
The large quadrilateral shown has two right angles, and two of the sides have lengths 7 cm and 10 cm. Points B and D are on the remaining two sides so that AB = 2 cm and DC = 6 cm. Calculate the area of the shaded quadrilateral.

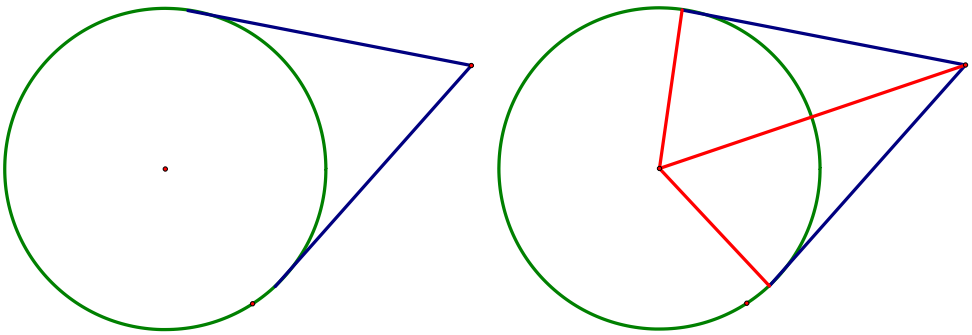


In the diagram, ABCD is a rectangle. Calculate the area of rectangle EGFH.

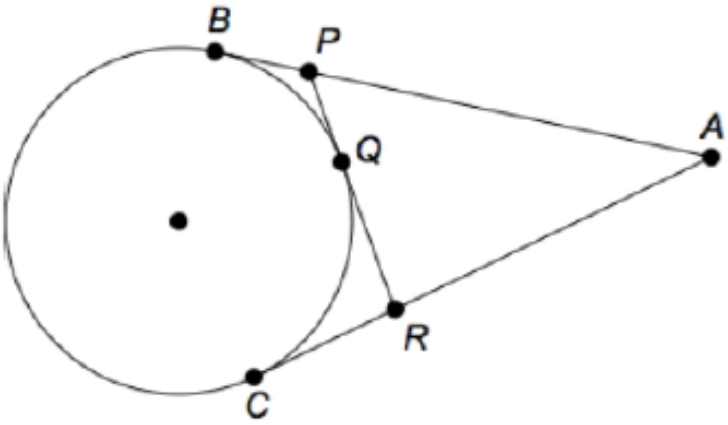
Tangent to a circle







Two tangents are drawn to a circle from a point A, which lies outside the circle; they touch the circle at points B and C respectively. A third tangent intersects AB in P and AC in R, and touches the circle at Q. If AB = 20, find the perimeter of triangle APR.



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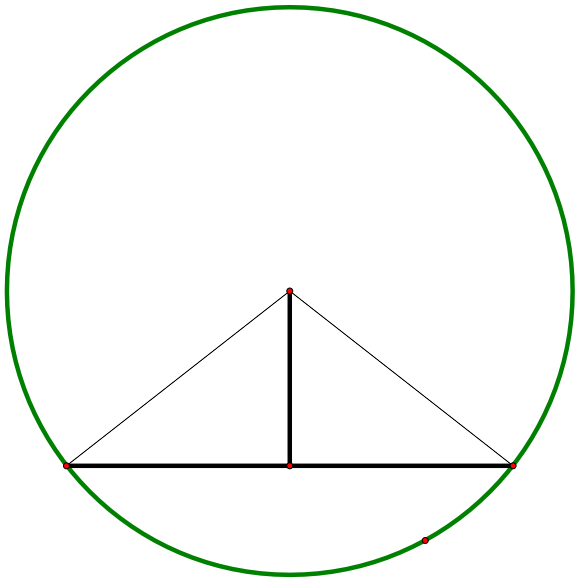
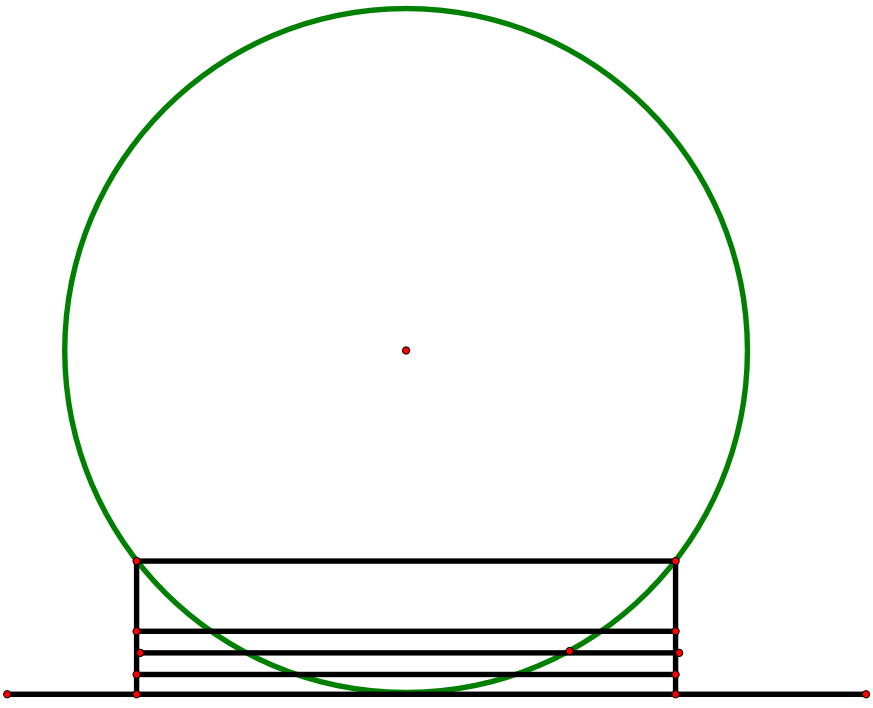
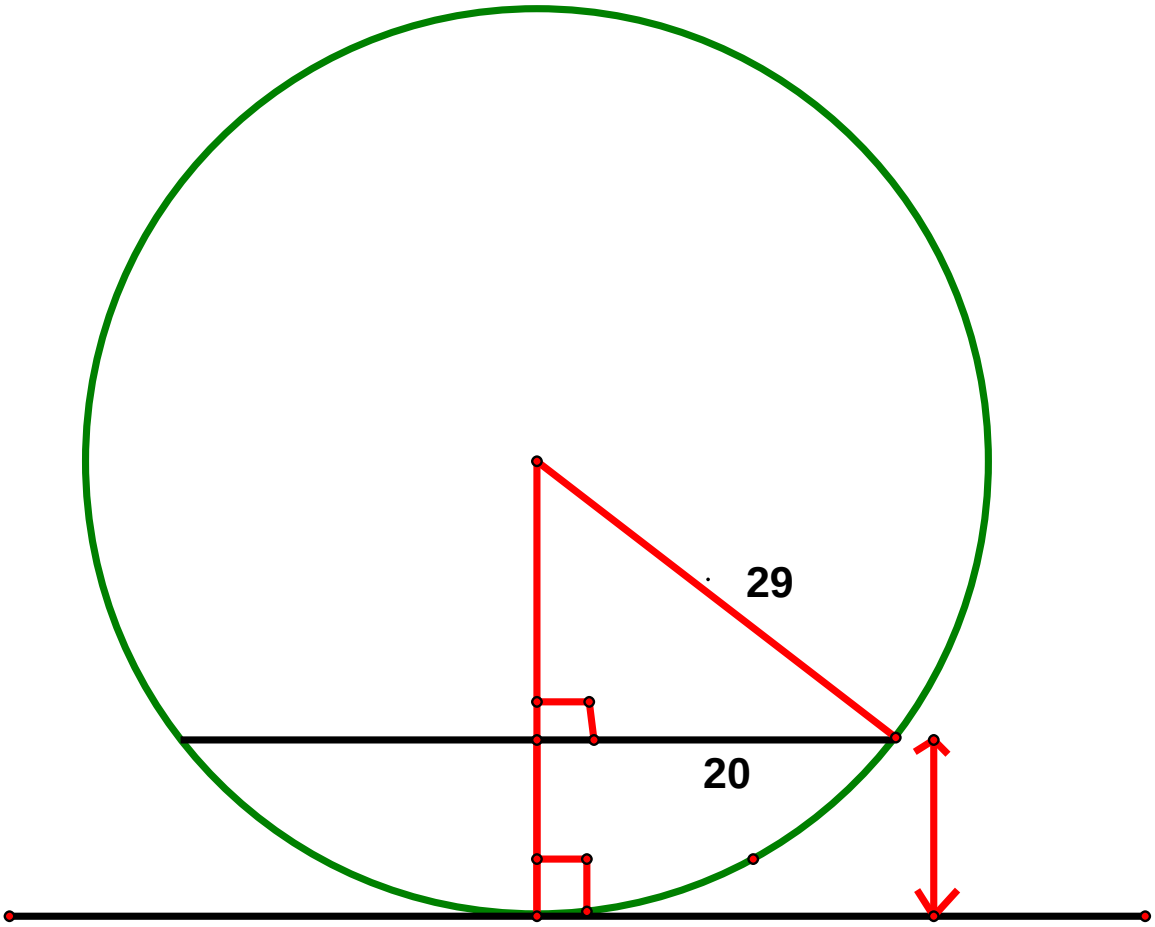


A certain type of ring has outer diameter of 58mm, an inner diameter of 40mm and thickness of 1mm. If one stacks enough rings on top of each other, it is possible to stand another ring vertically on top of the pile in such a way that the ring just touches the bottom. How many rings are in the stack when this happens?

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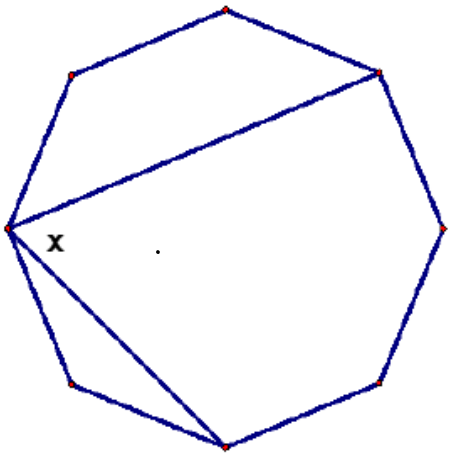
20

Polygons

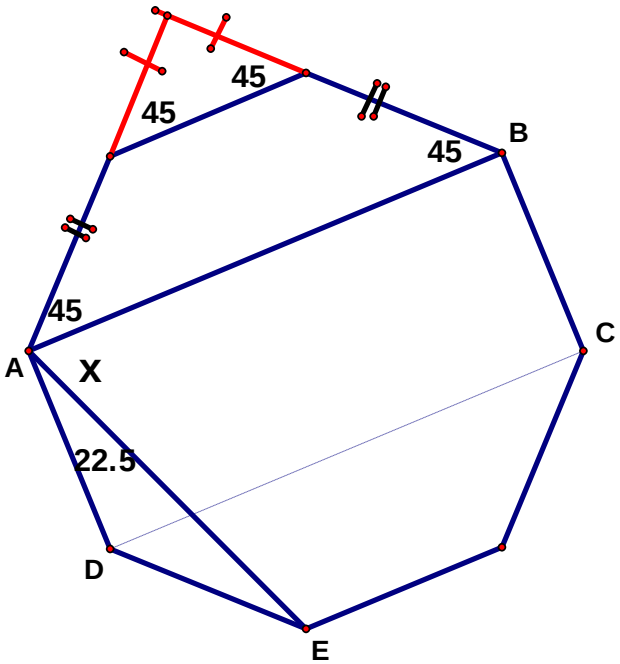
The sum of the exterior angles of any polygon is 360°.

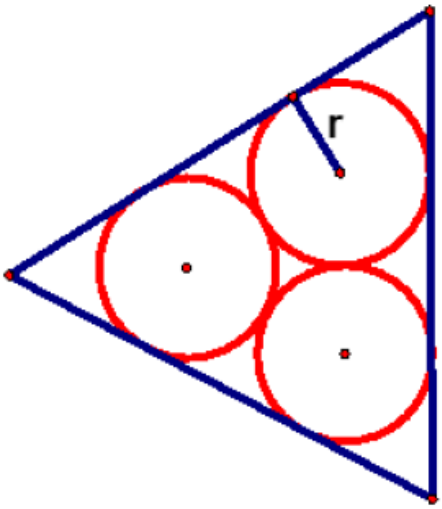
A regular polygon is one where all the sides and all the angles are equal.

Each exterior angle of a regular octagon is $\frac{360}{8} = 45^\circ$. So each interior angle is $180^\circ - 45^\circ = 135^\circ$

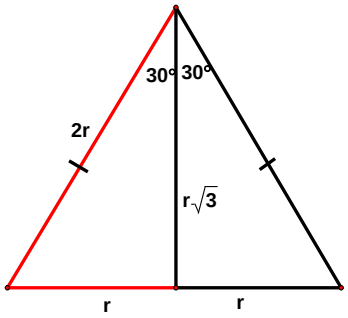


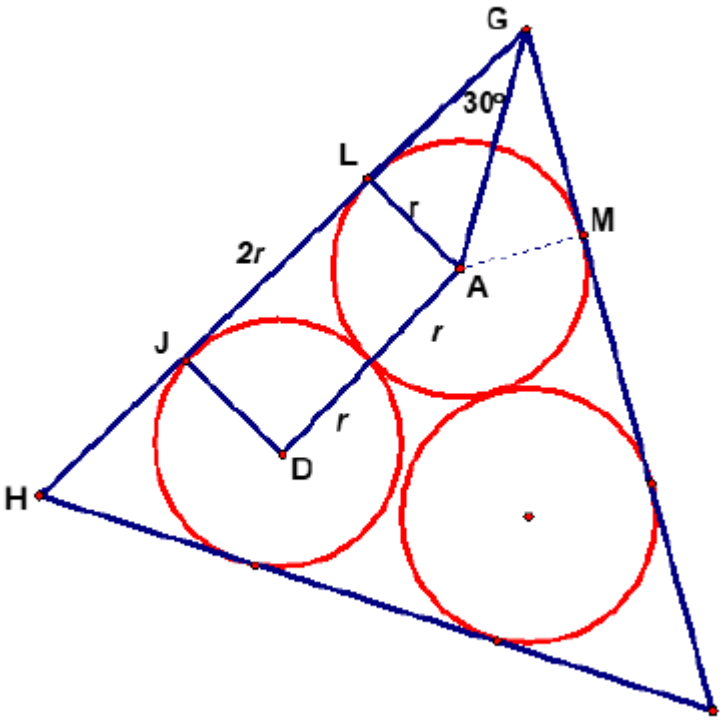
Determine the angle *x* in the regular octagon alongside.

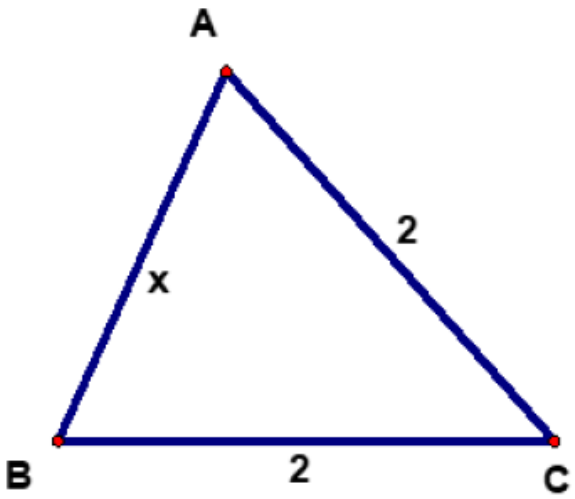




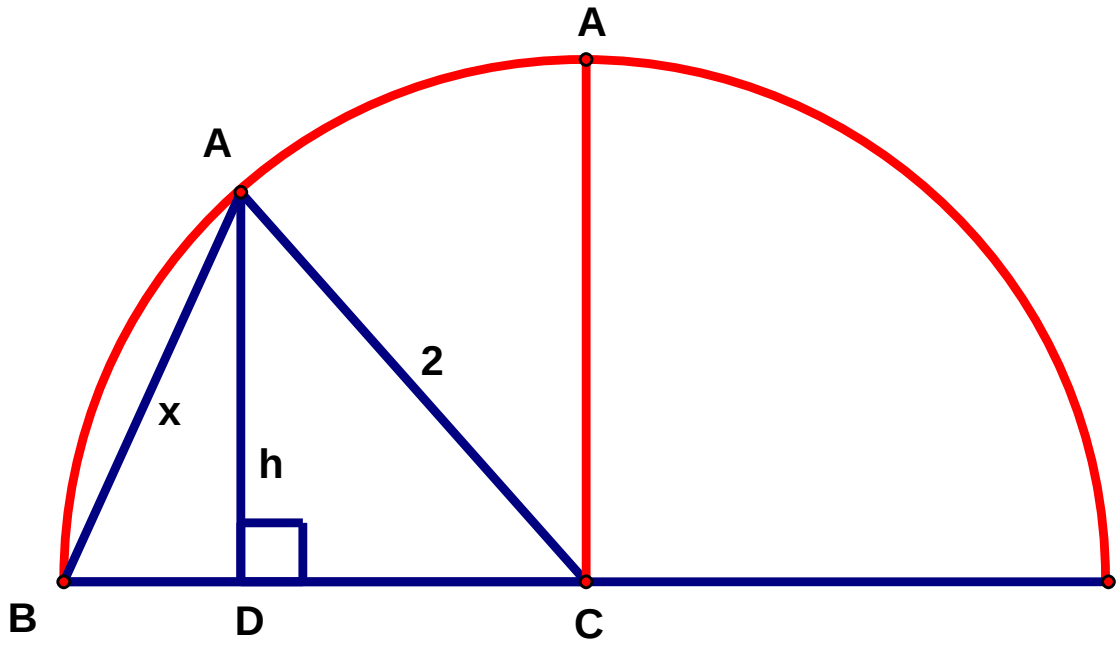
Three circles, each of radius *r*, are inscribed in an equi-lateral triangle with sides of length 1. If each circle touches the other two, as well as two of the sides of the triangle, calculate *r*.







An isosceles triangle has sides of length 2, 2 and x. For which value of x is the area of the triangle a maximum?



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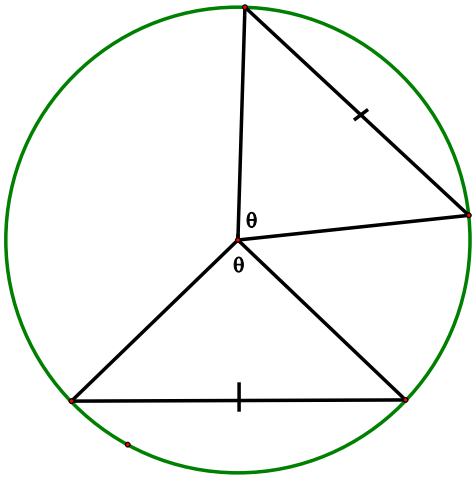
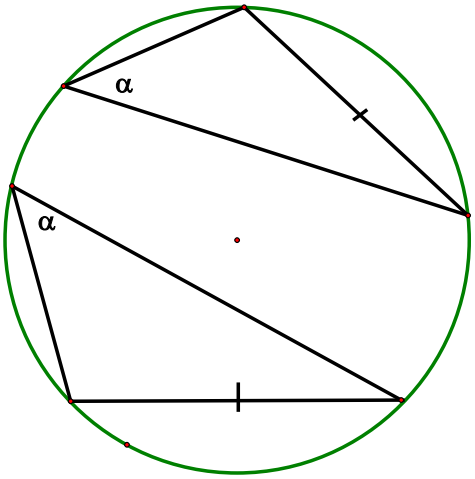
Circle Theorems

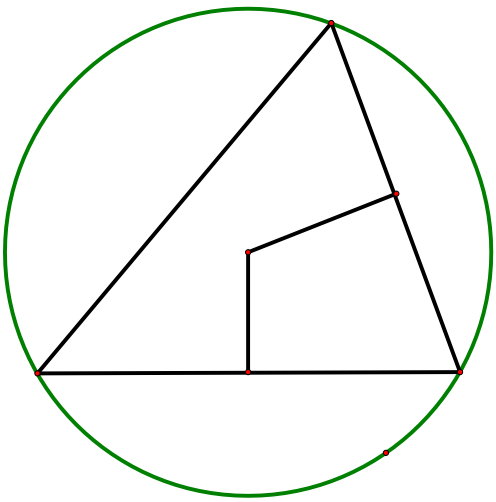
1. Equal chords subtend equal angles at the circumference





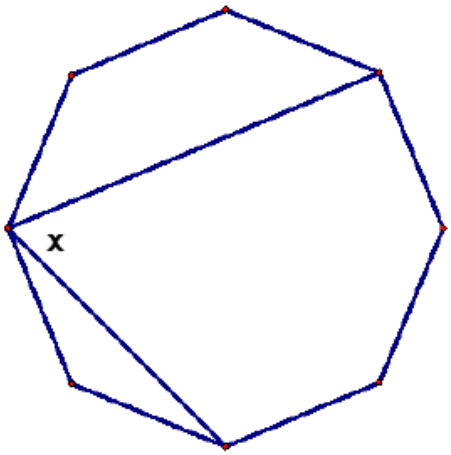
Cyclic quads

1. There is always a circle that passes through the vertices of a triangle



2. No such guarantee for four or more points.

3. For a quadrilateral, it can happen that the four vertices lie on a circle. When this happens we say the quadrilateral is cyclic. Likewise for more than four points.
4. **A regular polygon is always cyclic.**



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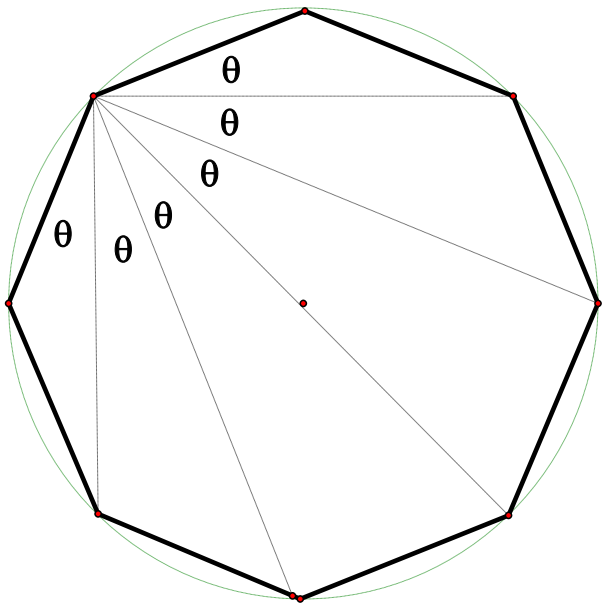
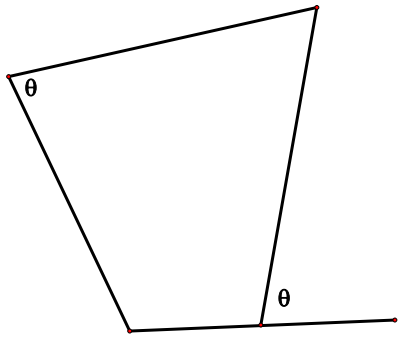
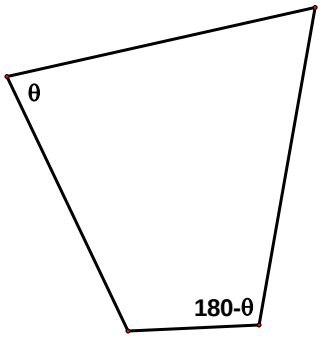
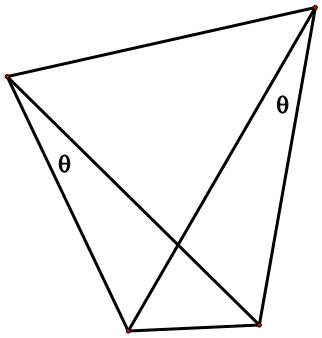


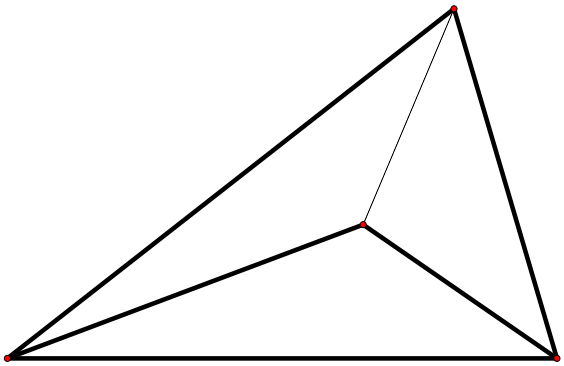
5. When any one of the following happens, the quadrilateral is cyclic.







6. The angle bisectors of a triangle meet at a point.



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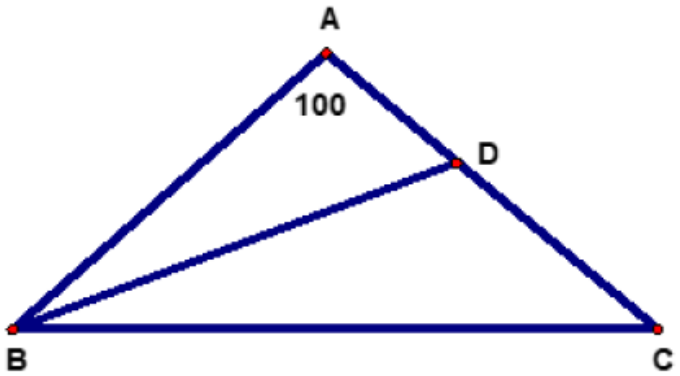
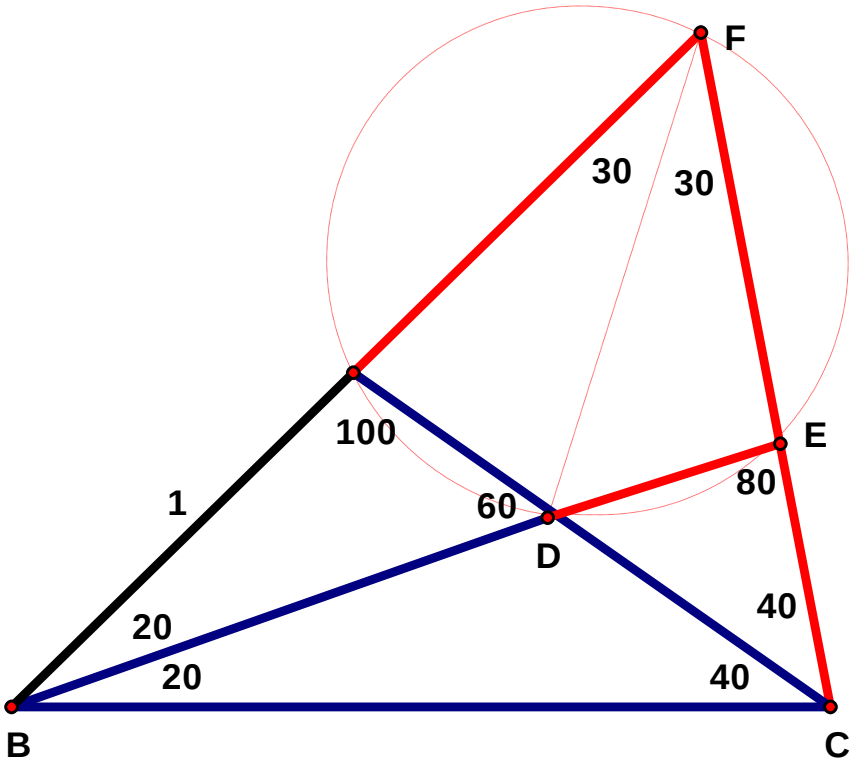
In ΔABC, $AB = AC$ and $\angle BAC = 100°$. The bisector of angle B meets AC at D. Prove that $AD + DB = BC$.

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40
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A hexagon is inscribed in a circle. Three of the sides have length a and three have length b. Determine the radius of the circle in terms of a and b.