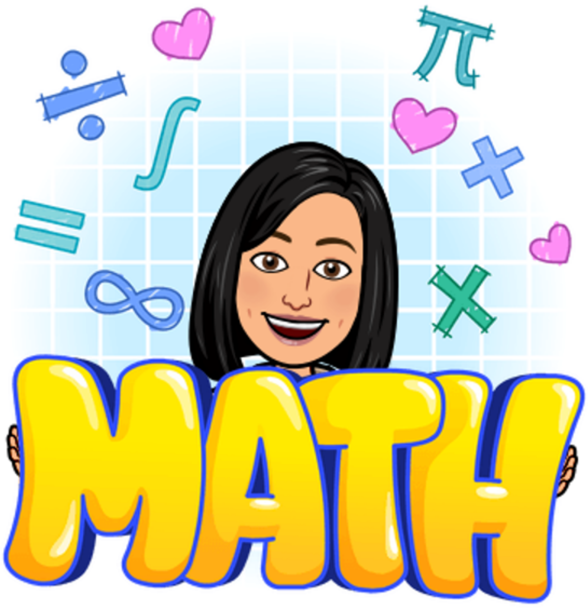

Welcome to Term One revision with Dr Rathilal

National Institute for Theoretical and Computational Sciences

Paper 1 (term 1)

1. Basic arithmetic
   - factorisation
   - inequalities
   - simplification of expressions
2. Number patterns & series

Number patterns

arithmetic

geometric

quadratic

Quadratic

$T_1, T_2, T_3, T_4, \ldots$

→ general term, T_n

1. $T_n = an^2 + bn + c, \ n \in \mathbb{N}$

$a = ?$

$b = ?$

$c = ?$

Yes

2. Has common 2nd difference

$a+b+c \rightarrow 72, \ 100, \ 120, \ 132, \ldots$

$3a+b \rightarrow [28, \ 20, \ 12, \ldots]$ (1st diff)

$2a \rightarrow -8 \quad -8$ (2nd diff)

Have $T_n = an^2 + bn + c$

$T_1 = a + b + c$

$T_2 = 4a + 2b + c$

$T_3 = 9a + 3b + c$

$T_4 = 16a + 4b + c$

T_1 ; T_2 ; T_3 ; T_4

$a + b + c$, $4a + 2b + c$, $9a + 3b + c$, $16a + 4b + c$

1st diff: $3a + b$, $5a + b$, $7a + b$

$2a$, $2a$

- 2nd diff $= 2a$
- first term in the 1st diff sequence is $3a + b$
- $T_1 = a + b + c$

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 20 = 0$ (3)

1.1.2 $3x^2 - 2x - 6 = 0$ (correct to TWO decimal places) (4)

1.1.3 $(x-1)^2 > 9$ (4)

1.1.4 $2\sqrt{x+6} + 2 = x$ (4)

1.2 Solve simultaneously for x and y:

$4x + y = 2$ and $4x + y^2 = 8$ (5)

1.3 If it is given that $2^x \times 3^y = 24^6$, determine the numerical value of $x - y$. (4)

[24]

QUESTION 2

2.1 Consider the quadratic sequence: 72 ; 100 ; 120 ; 132 ; … $T_n = an^2 + bn + c$

2.1.1 Determine T_n, the n^{th} term of the quadratic sequence. (4)

$T_n = K_{12}$

$T_n = -60$

2.1.2 A term in the quadratic sequence 72 ; 100 ; 120 ; 132 ; … is equal to the twelfth term of the sequence of first-differences. Determine the position of this term in the quadratic sequence. (5)

T_n

K_{12}

arithmetic sequence

Q2.1.1. Find T_n

Find a, b and c.

$-8 = 2a \Rightarrow a = -4$

$3a + b = 28$

$3(-4) + b = 28$

$b = 28 + 12 = 40$

$a + b + c = 72$

$-4 + 40 + c = 72$

$C = 72 - 36 = 36$

so $T_n = an^2 + bn + C$

$T_n = -4n^2 + 40n + 36, \quad n \in \mathbb{N}$

Arithmetic sequence

$T_1, \; T_2, \; T_3, \ldots$

$(T_2 - T_1) = (T_3 - T_2) = d$

$a = T_1, \; T_2, \; T_3, \; T_4, \; T_5, \ldots$

$d \quad d \quad d \quad d$

$a + d + d + d$

$T_n = a + (n-1)d$

Q 2.1.2

first diff sequence : $28, \; 20, \; 12, \ldots$

$-8 \quad -8$

Lets denote the general term of the above sequence by K_n

$$K_n = a + (n-1)d$$
$$= 28 + (n-1)(-8)$$
$$K_n = 28 - 8n + 8$$
$$K_n = 36 - 8n$$

$$K_{12} = 36 - 8(12) = -60$$

Have $T_n = -4n^2 + 40n + 36$

Find n when

$$T_n = -60$$
$$-4n^2 + 40n + 36 = -60$$
$$-4n^2 + 40n + 96 = 0$$

$\div -4]$
$$n^2 - 10n - 24 = 0$$
$$(n-12)(n+2) = 0$$

$n = 12$ ✓ or ~~$n = -2$~~ ✗

$$T_{12} = K_{12}$$

S_n = sum to n terms

$S_n =$

$a \approx T_1, T_2, T_3 \mid r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$

ratio $= r$

QUESTION 3 Geometric Sequences

$T_n = ar^{n-1}$

3.1 If $r = \frac{1}{5}$ and $a = 2\ 000$, determine:

3.1.1 T_n, the general term of the series (1)

3.1.2 T_7 (1)

3.1.3 Which term of the series will have a value of $\frac{16}{15\ 625}$ $S_n = \frac{16}{15625}$ (3)

3.2 Consider the geometric series where $\sum_{n=1}^{\infty} T_n = 27$ and $S_3 = 26$.

Calculate the value of the constant ratio (r) of the series. (4)
[9]

QUESTION 4

The lines $y = x + 1$ and $y = -x - 7$ are the axes of symmetry of the function $f(x) = \frac{-2}{x+p} + q$.

4.1 Show that $p = 4$ and $q = -3$. (4)

4.2 Calculate the x-intercept of f. (2)

4.3 Sketch the graph of f. Clearly label ALL intercepts with the axes and the asymptotes. (4)
[10]

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

arith metic seq ✓ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$ ✓ Derivation

Geometric $T_n = ar^{n-1}$ ✓ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ ✓ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$ ✓ Sum to infinity

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$