NITheCS
National Institute for Theoretical and Computational Sciences

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

S E M I N A R

Two interesting applications of undergraduate algebra to Number Theory



Prof Poobhalan Pillay
(University of KwaZulu-Natal)

ABSTRACT

Among the extensions of $\mathbb{Z}$ in $\mathbb{R}$, or even the complex field, are the quadratic $\mathbb{Z}[\sqrt{N}]$ extensions $\mathbb{Z}$ for positive non-square integers N, and $\mathbb{Z}$. All of these are PIDs (principal ideal domains that are rings in which the division algorithm $n = qm + r$ holds).

These are very strong commutative rings. They are not fields, but their invertible elements are of independent interest. All PIDs admit unique factorisations, but UFDs are not necessarily PIDs. Striking examples are $\mathbb{Z}[x]$ and $\mathbb{Z}[\sqrt{-5}]$.

This talk will deal with the units of $\mathbb{Z}[\sqrt{N}]$ and their connections to Fibonacci sequences and the two Pell equations, $x^2 - Ny^2 = \pm 1$, where x and y are integers. A by-product of this investigation is to provide rational approximations to square roots of positive integers.

If time permits, I will demonstrate how field theory can be used to prove that $4n + 1$ primes can always be written as sums of two integral squares.

Prof Poobhalan Pillay is an Emeritus Professor of Mathematics at the University of KwaZulu-Natal (UKZN). His research interests lie in algebra, with a strong interest in Euclidean geometry and elementary number theory. Since retiring from active service in 2006, he has devoted much of his time to training talented high school learners to develop their mathematical problem-solving skills.

He has authored and co-authored several books, including 'Classification of Commutative FPF Rings', a research monograph co-authored with the late Prof Carl Faith of Rutgers University, and '1000 Mathematics Olympiad Problems, Complete with Lessons and Solutions', a 643-page manual that has gained popularity worldwide.

Date:
Thursday, 5 November 2026

Time:
12h15-13h15 SAST

Venues:

- Computer Science Seminar Room (Room 305)
  University of KwaZulu-Natal
  Westville Campus
  3rd Floor, H-Block,
- Online

WHO SHOULD ATTEND?
All are welcome.

ENQUIRIES:
Email Ms Laila Gurudas:
Gurudas@ukzn.ac.za

REGISTER TO ATTEND:

science, technology & innovation
Department:
Science, Technology and Innovation
REPUBLIC OF SOUTH AFRICA

NRF National Research Foundation